CS 343: Artificial Intelligence

CSPs II + Local Search

Prof. Yuke Zhu

The University of Texas at Austin

[These slides based on those of Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]

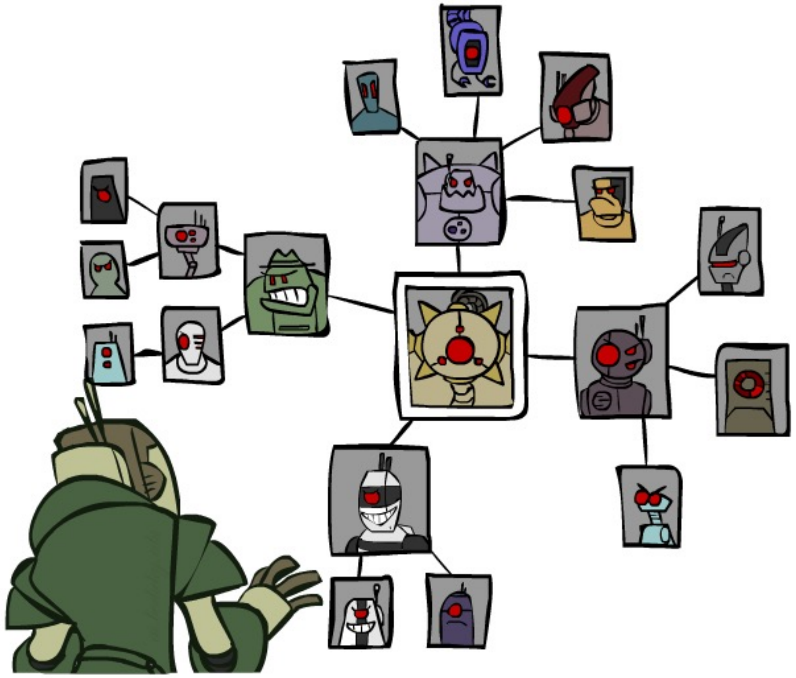
Announcements

- Homework 1: Search
  - Reminder: Due Monday 1/30 at 11:59 pm
- Project 1: Search
  - Reminder: Due Wednesday 2/1 at 11:59 pm
- Readings: Adversarial Search, Utilities
  - Textbook Chapters 5 (Sections 5.1-5.5) and 16 (Sections 16.1-16.3)
  - Due Monday 1/30, at 5:00 pm.
- Homework 2: CSPs, Games, Utilities
  - Has been released! Due Monday 2/13, at 11:59 pm.

Today

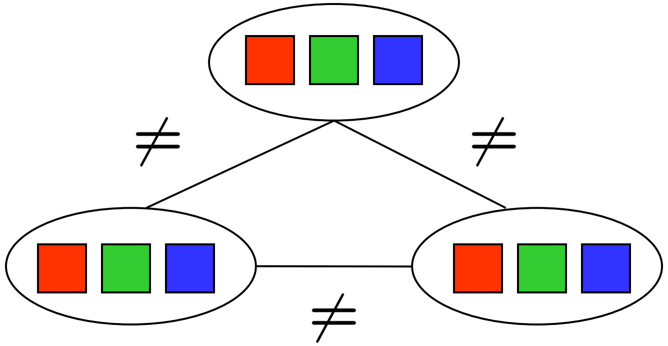
- Efficient Solution of CSPs

- Local Search

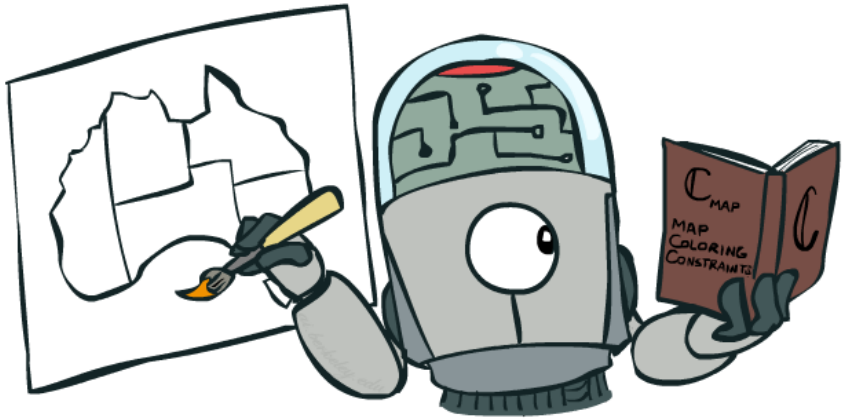


Last time: CSPs

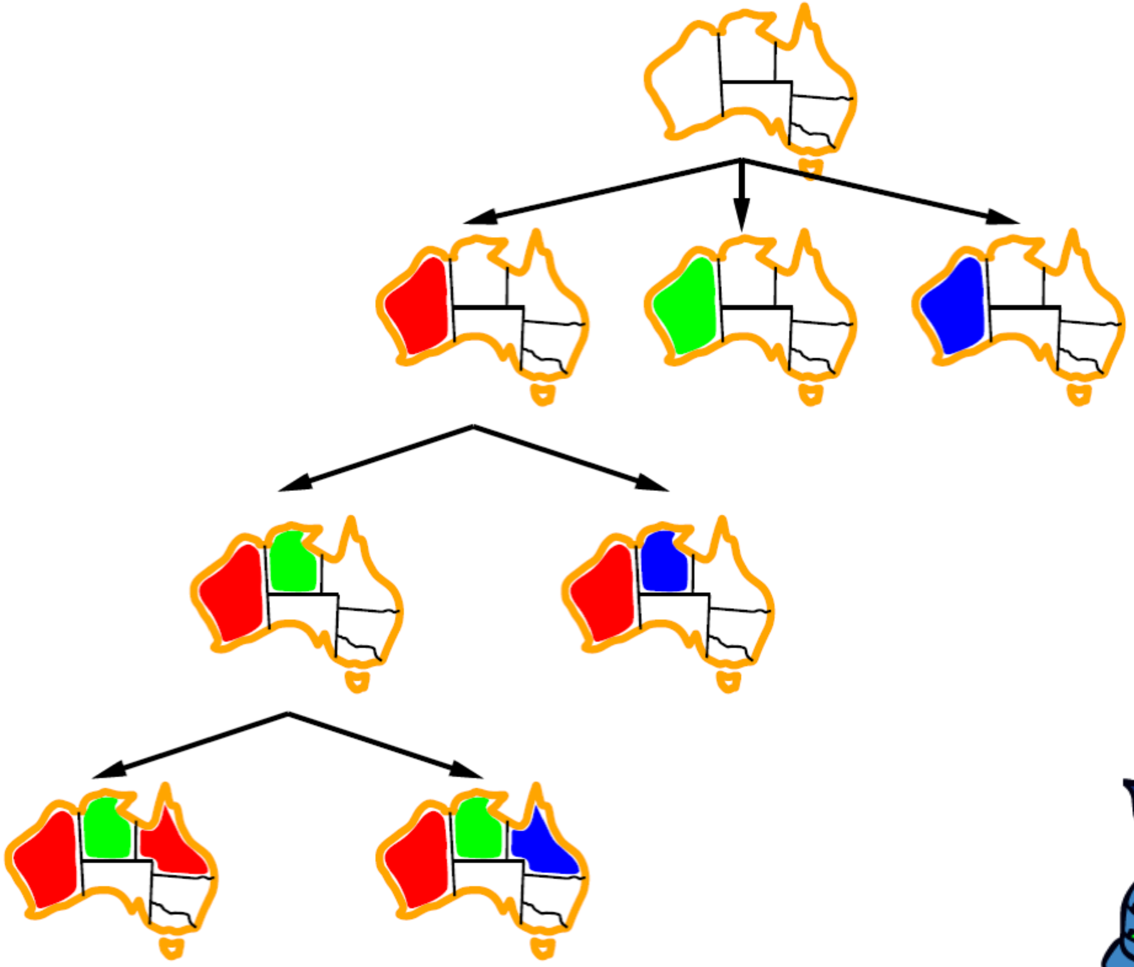
- CSPs:
  - Variables
  - Domains
  - Constraints
    - Implicit (provide code to compute)
    - Explicit (provide a list of the legal tuples)
    - Unary / Binary / N-ary

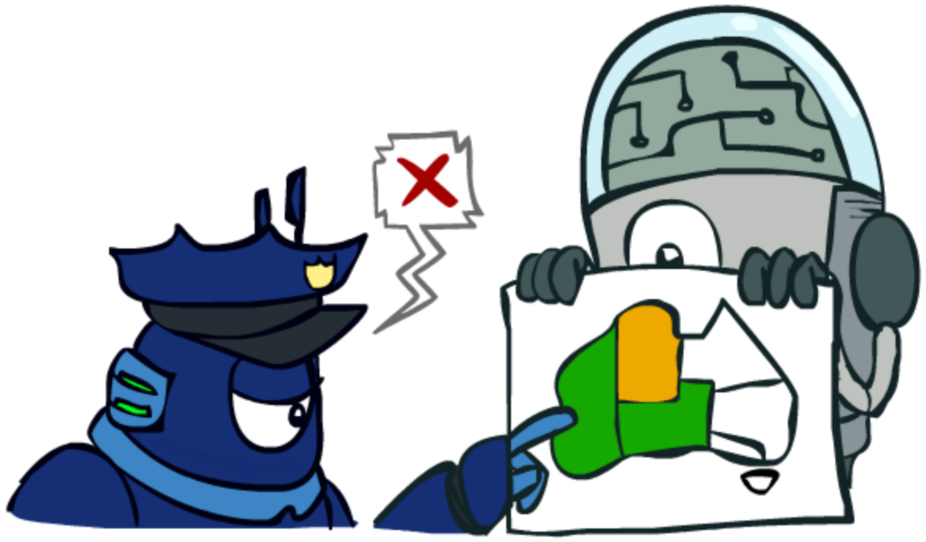


- Goals:
  - Here: find any solution
  - Also: find all, find best, etc.



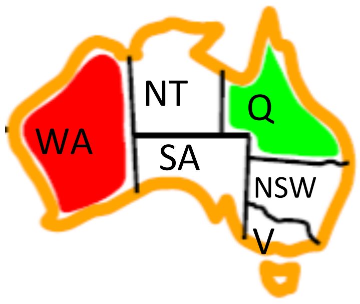
Last time: Backtracking

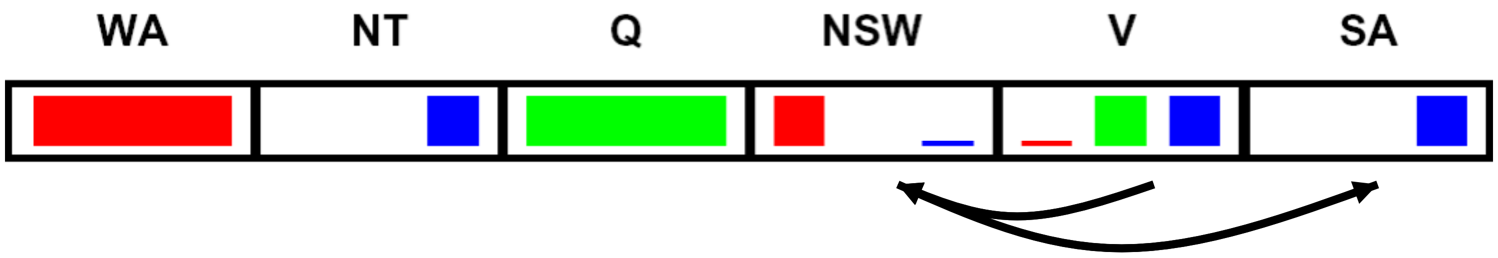




Last time: Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:





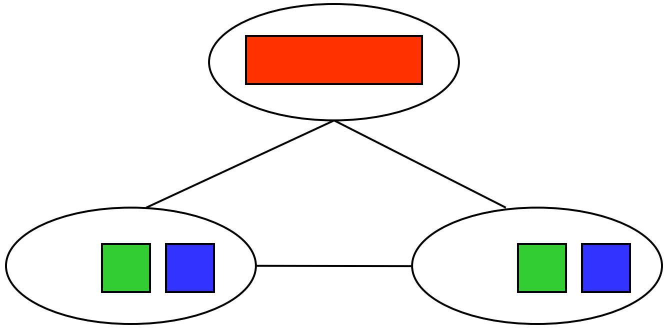
- Important: If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing arc consistency?

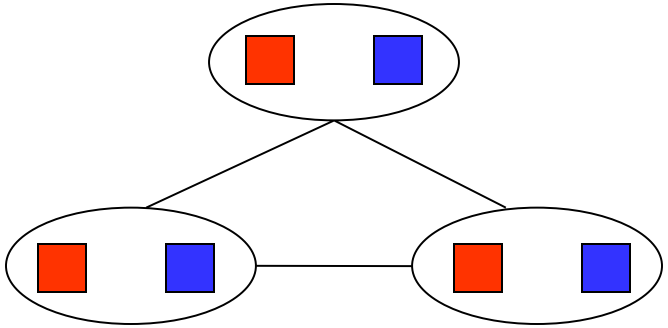
Remember: Delete from the tail!

Last time: Limitations of Arc Consistency

- After enforcing arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)

- Arc consistency still runs inside a backtracking search!





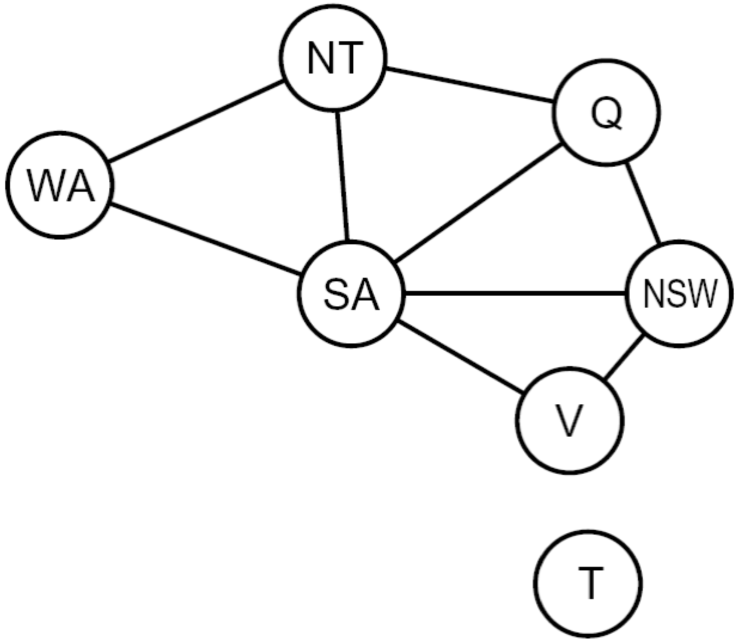
What went wrong here?

Last time: Improving Backtracking

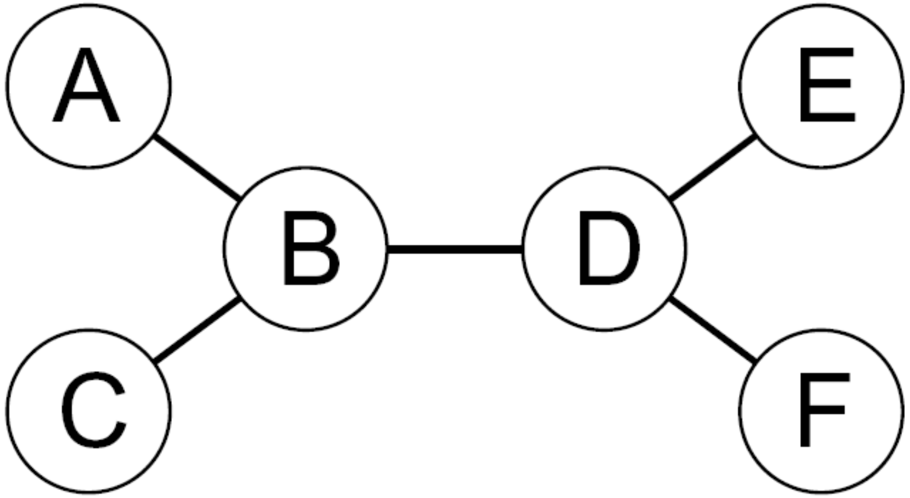
- General-purpose ideas give huge gains in speed
  - … but it's all still NP-hard
- Filtering: Can we detect inevitable failure early?
- Ordering:
  - Which variable should be assigned next? (MRV)
  - In what order should its values be tried? (LCV)
- Structure: Can we exploit the problem structure?

Problem Structure

- Extreme case: independent subproblems
  - Example: Tasmania and mainland do not interact
- Independent subproblems are identifiable as connected components of constraint graph
- Suppose a graph of n variables can be broken into subproblems of only c variables:
  - Worst-case solution cost is $O((n/c)(d^c))$, linear in n
  - E.g., n = 80, d = 2, c =20
  - 2^{80} = 4 billion years at 10 million nodes/sec
  - $(4)(2^{20})$ = 0.4 seconds at 10 million nodes/sec



Tree-Structured CSPs

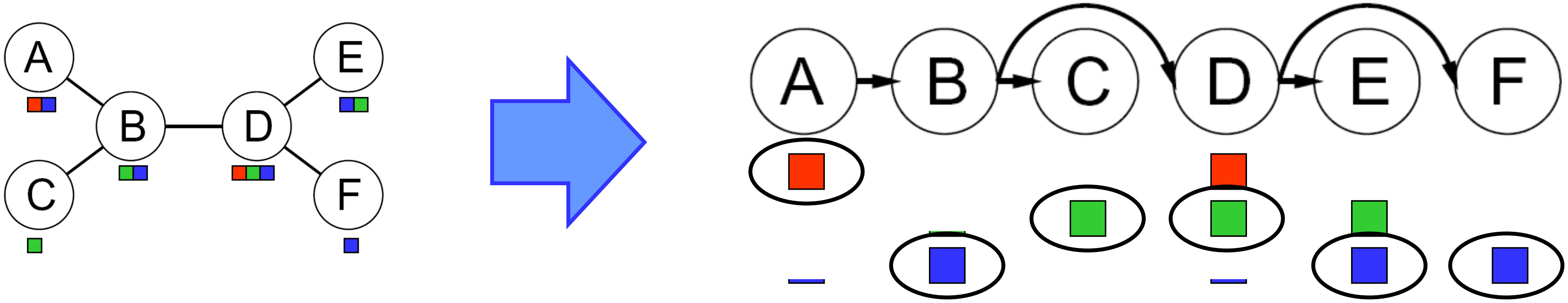


- Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n\ d^2)$ time
  - Compare to general CSPs, where worst-case time is $O(d^n)$

- This property also applies to probabilistic reasoning (later): an example of the relation between syntactic restrictions and the complexity of reasoning

Tree-Structured CSPs

- Algorithm for tree-structured CSPs:
  - Order: Choose a root variable, order variables so that parents precede children

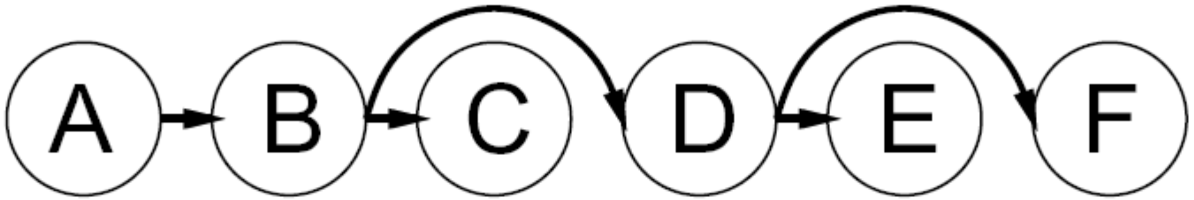


  - Remove backward: For i = n : 2, apply RemoveInconsistent(Parent(X_i),X_i)
  - Assign forward: For i = 1 : n, assign X_i consistently with Parent(X_i)

- Runtime: $O(n\ d^2)$ (why?)

Tree-Structured CSPs

- Claim 1: After backward pass, all root-to-leaf arcs are consistent
- Proof: Each X→Y was made consistent at one point and Y's domain could not have been reduced thereafter (because Y's children were processed before Y)

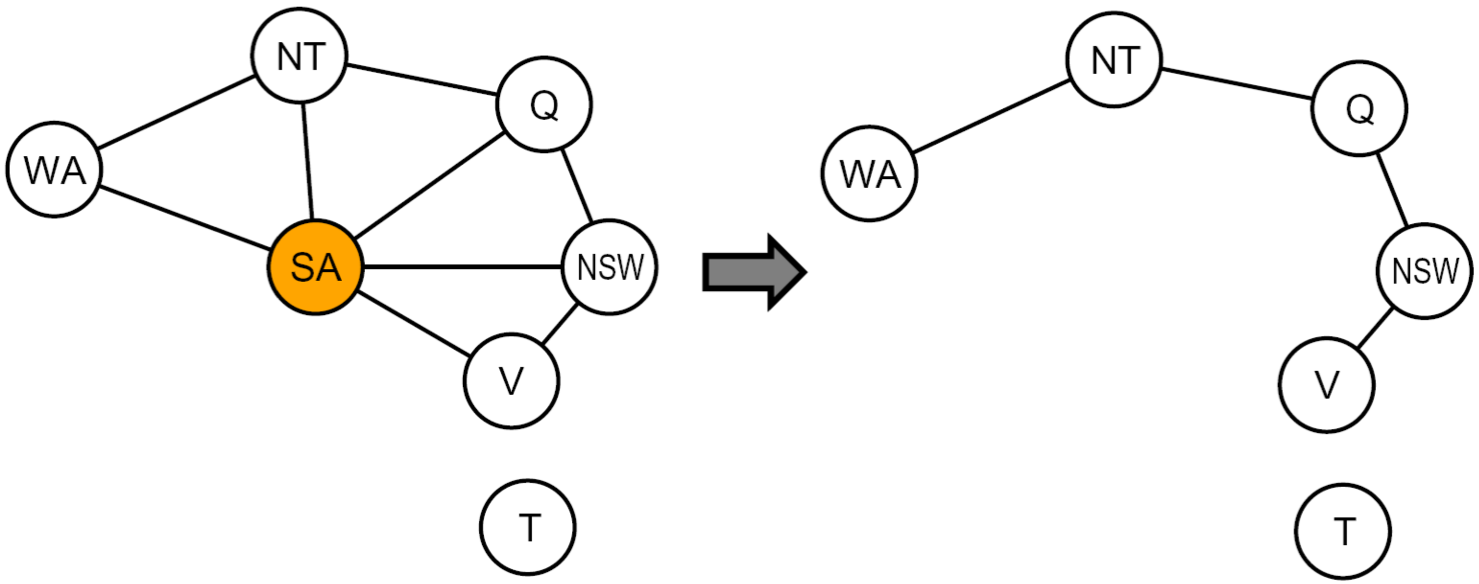


- Claim 2: If root-to-leaf arcs are consistent, forward assignment will not backtrack
- Proof: Induction on position

- Why doesn't this algorithm work with cycles in the constraint graph?

- Note: we'll see this basic idea again with Bayes' nets

Nearly Tree-Structured CSPs



- Conditioning: instantiate a variable, prune its neighbors' domains
- Cutset conditioning: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree
- Cutset size c gives runtime $O((d^c) (n-c) d^2)$, very fast for small c

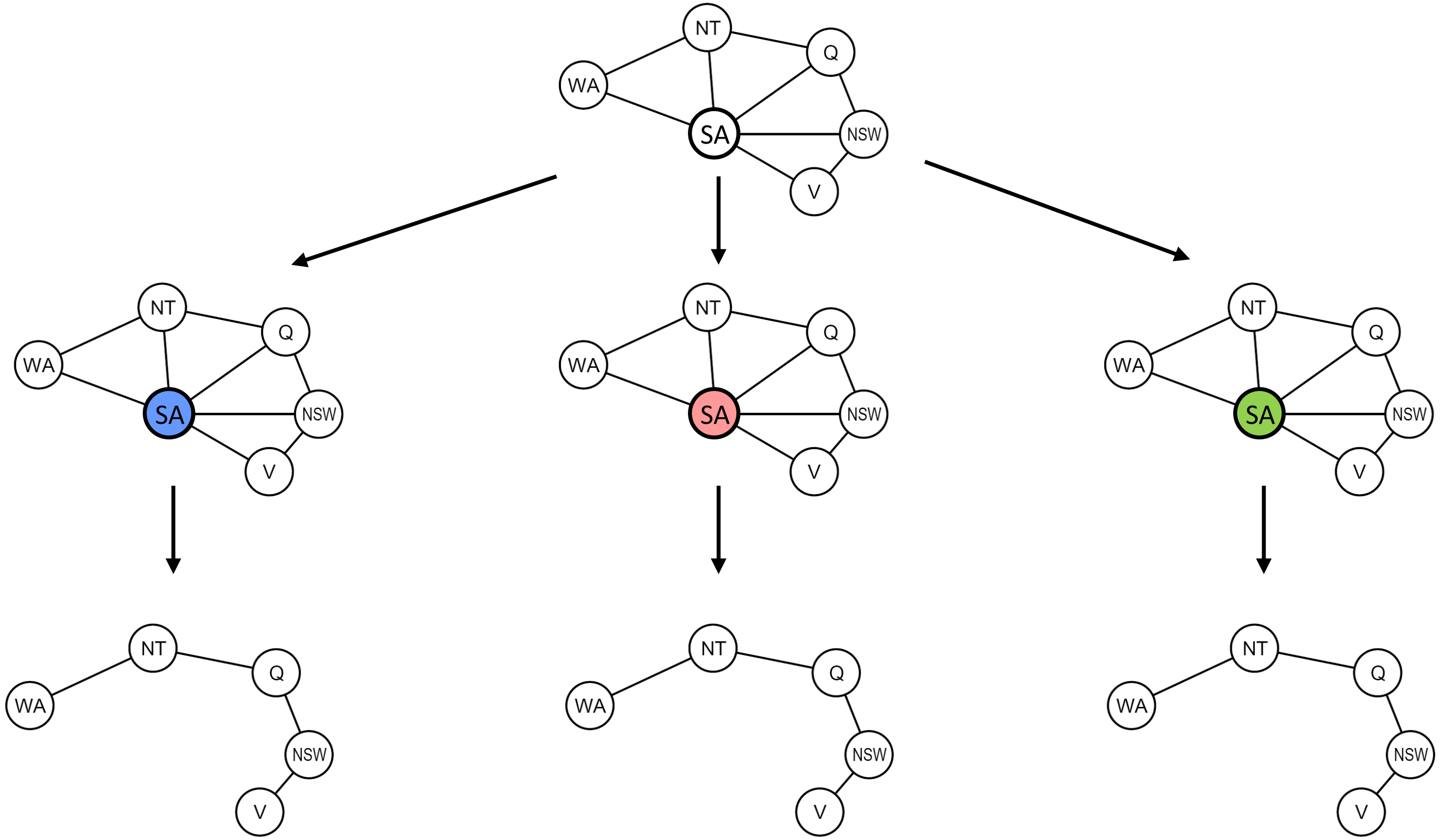
Cutset Conditioning

Choose a cutset

Instantiate the cutset (all possible ways)

Compute residual CSP for each assignment

Solve the residual CSPs (tree structured), removing any inconsistent domain values w.r.t. cutset assignment



d^c

$(n\text{-}c)d^2$

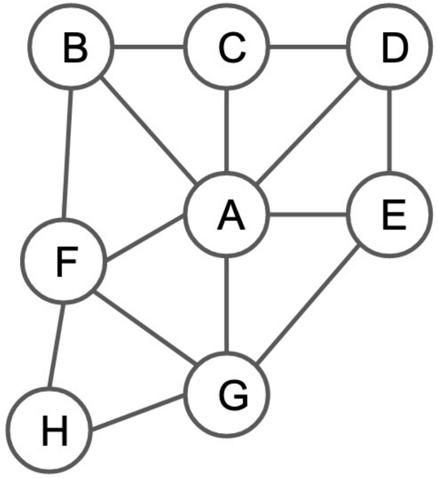
Exercise: Cutset Exercise

Tree-structured CSP

Now we want to color all nodes in the graph with color Red, Green, Blue and Yellow. Any pair of connected nodes cannot have the same color.

(1) How many minimal cutsets are there? What are they? (Note: a minimal cutset is a cutset with the smallest number of nodes)

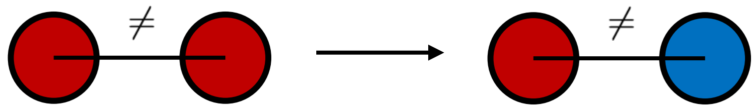
(2) How many residual tree-structured CSPs do we have after cutset conditioning?



Iterative Algorithms for CSPs

- Local search methods typically work with "complete" states, i.e., all variables assigned

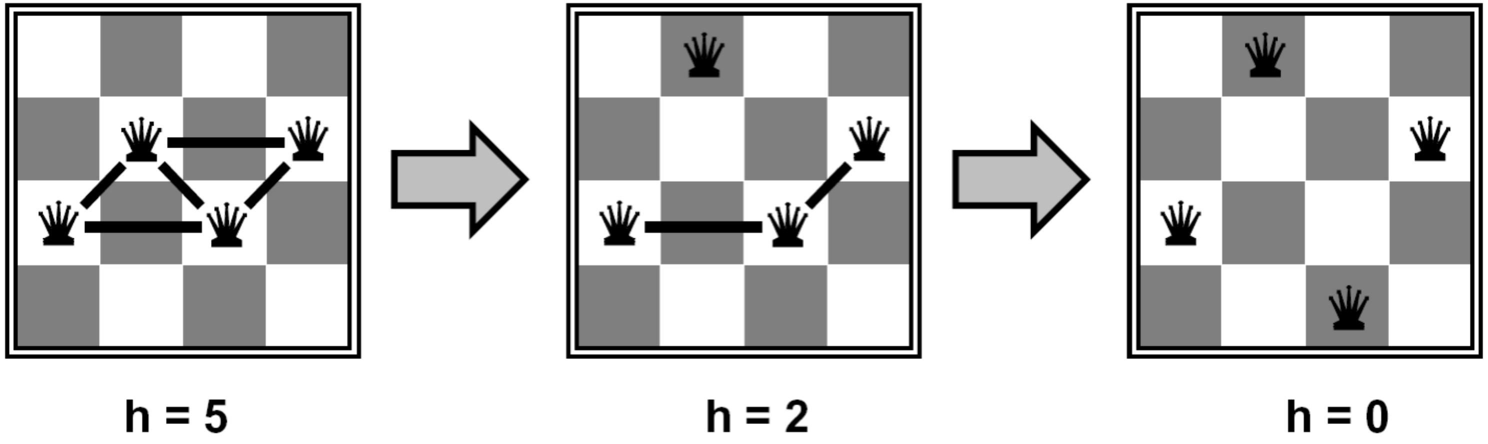
- To apply to CSPs:
  - Take an assignment with unsatisfied constraints
  - Operators *reassign* variable values
  - No fringe! Live on the edge.



- Algorithm: While not solved,
  - Variable selection: randomly select any conflicted variable
  - Value selection: min-conflicts heuristic:
    - Choose a value that violates the fewest constraints
    - i.e., hill climb with h(n) = total number of violated constraints

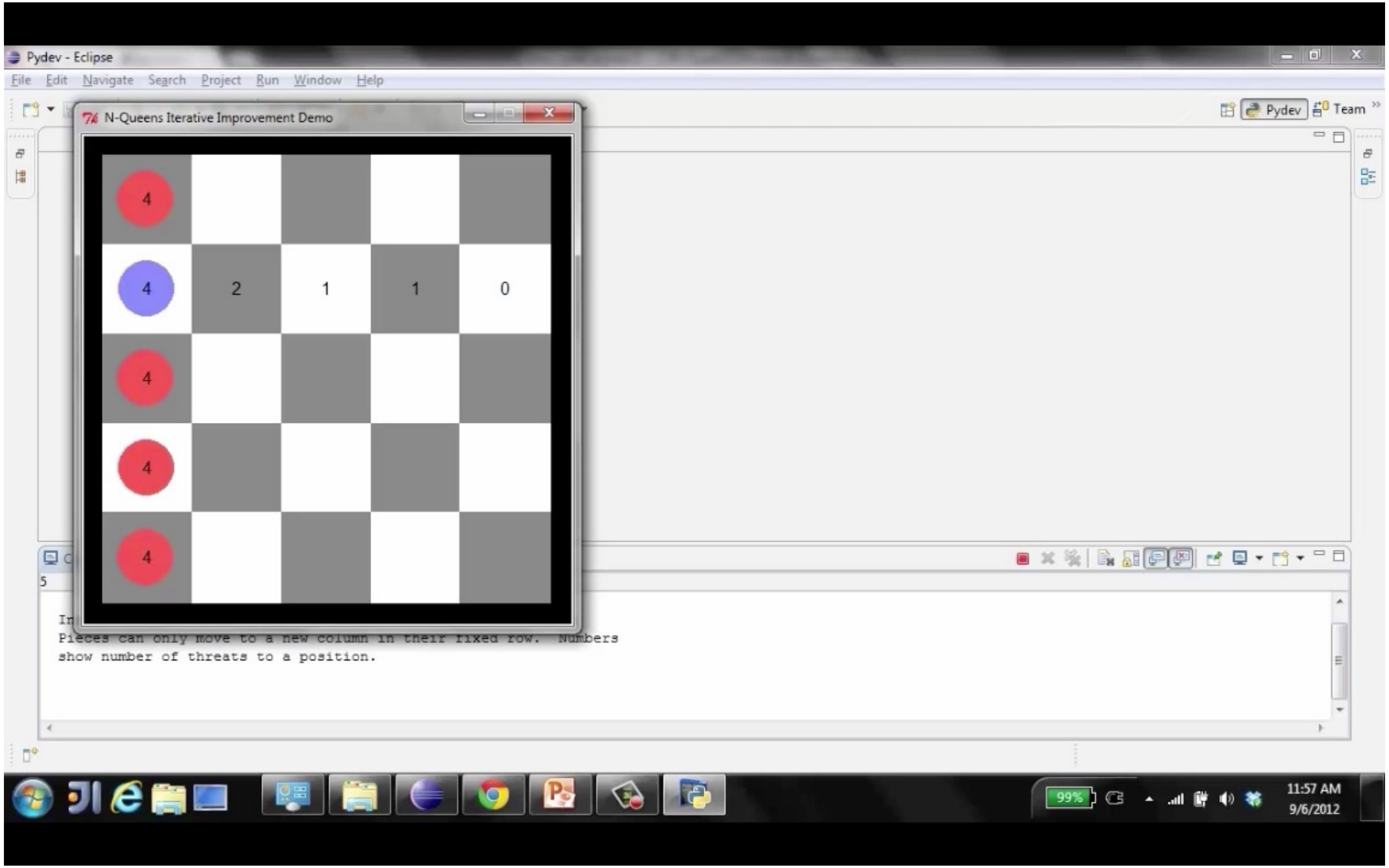
- Can get stuck in local minima (we'll come back to this idea in a few slides)

Example: 4-Queens



- States: 4 queens in 4 columns (4^4 = 256 states)
- Operators: move queen in column
- Goal test: no attacks
- Evaluation: c(n) = number of attacks

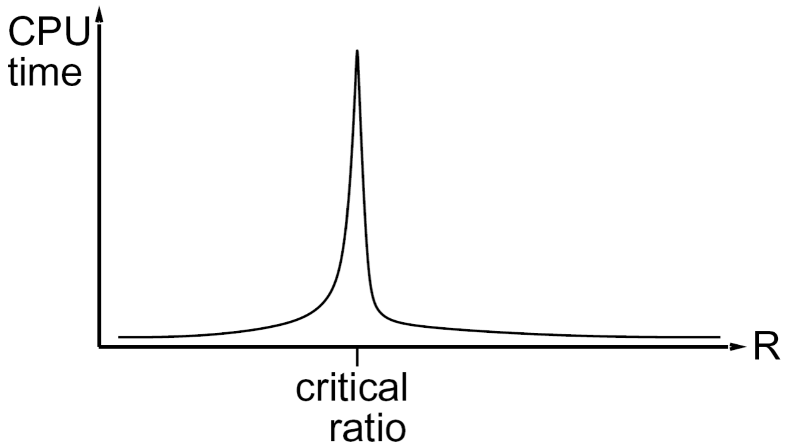
Video of Demo Iterative Improvement – n Queens

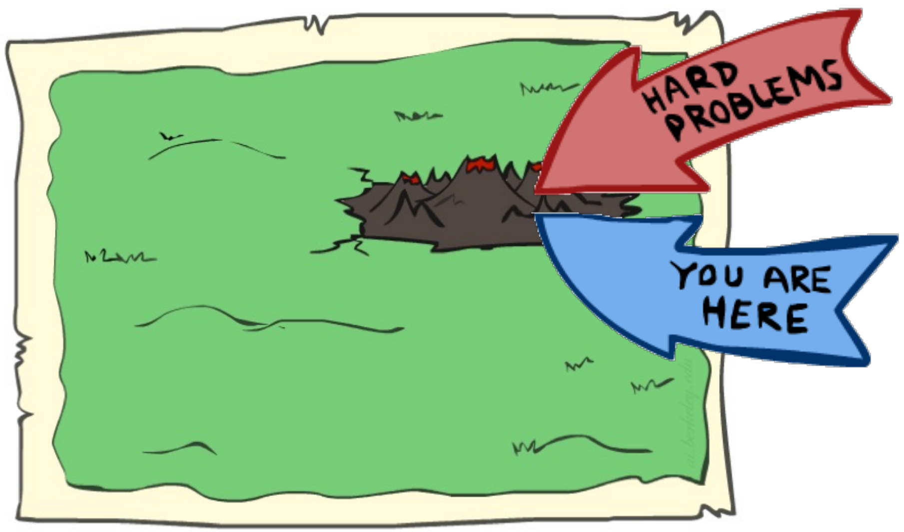


Performance of Min-Conflicts

- Runtime of min-conflicts is on n-queens is **roughly independent of problem size**!
  - Why?? Solutions are densely distributed in state space
- Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000) in ~50 steps!
- The same appears to be true for any randomly-generated CSP *except* in a narrow range of the ratio

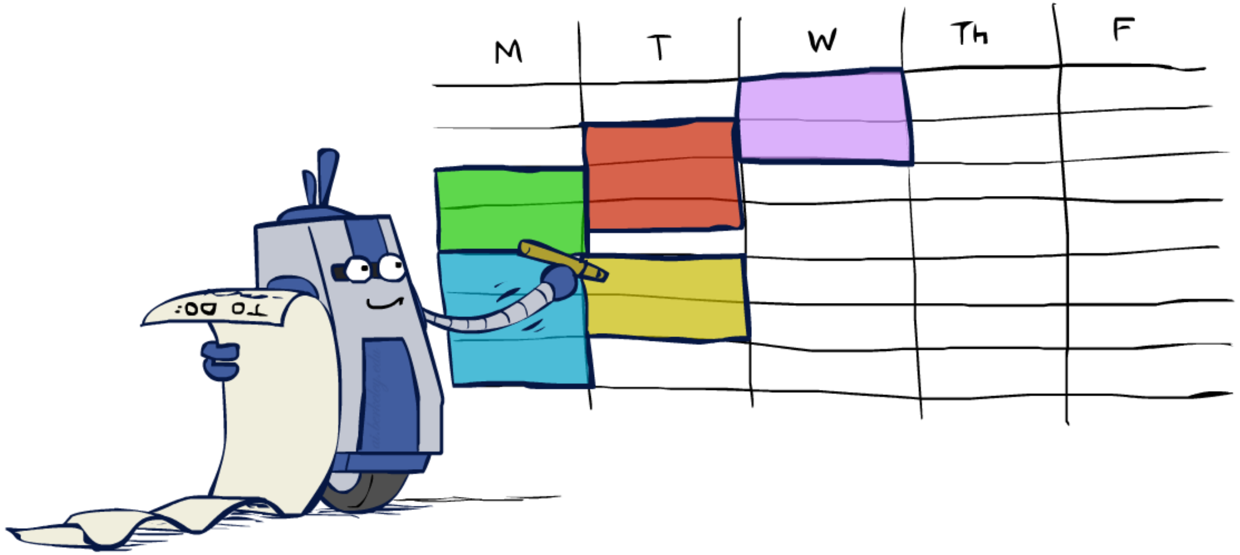
$$R = \frac{\text{number of constraints}}{\text{number of variables}}$$





Summary: CSPs

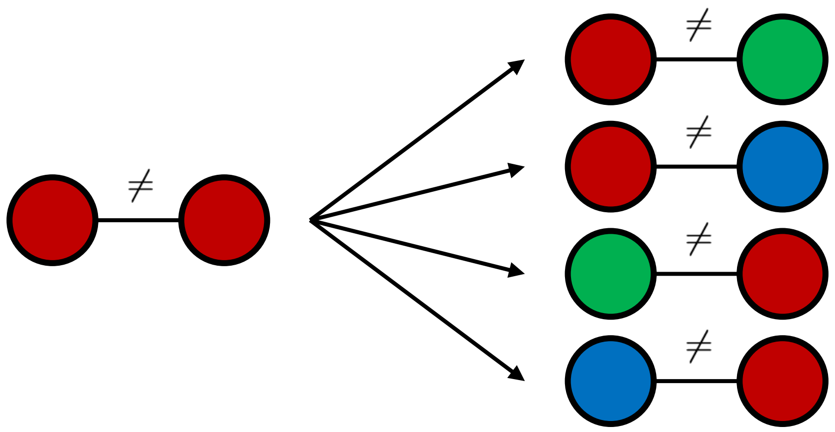
- CSPs are a special kind of search problem:
  - States are partial assignments
  - Goal test defined by constraints
- Basic solution: backtracking search
- Speed-ups:
  - Filtering
  - Ordering
  - Structure
- Iterative min-conflicts is often effective in practice



Local Search

Local Search

- Tree search keeps unexplored alternatives on the fringe (ensures completeness)
- Local search: improve a single option until you can't make it better (no fringe!)
- New successor function: local changes



- Generally much faster and more memory efficient (but incomplete and suboptimal)

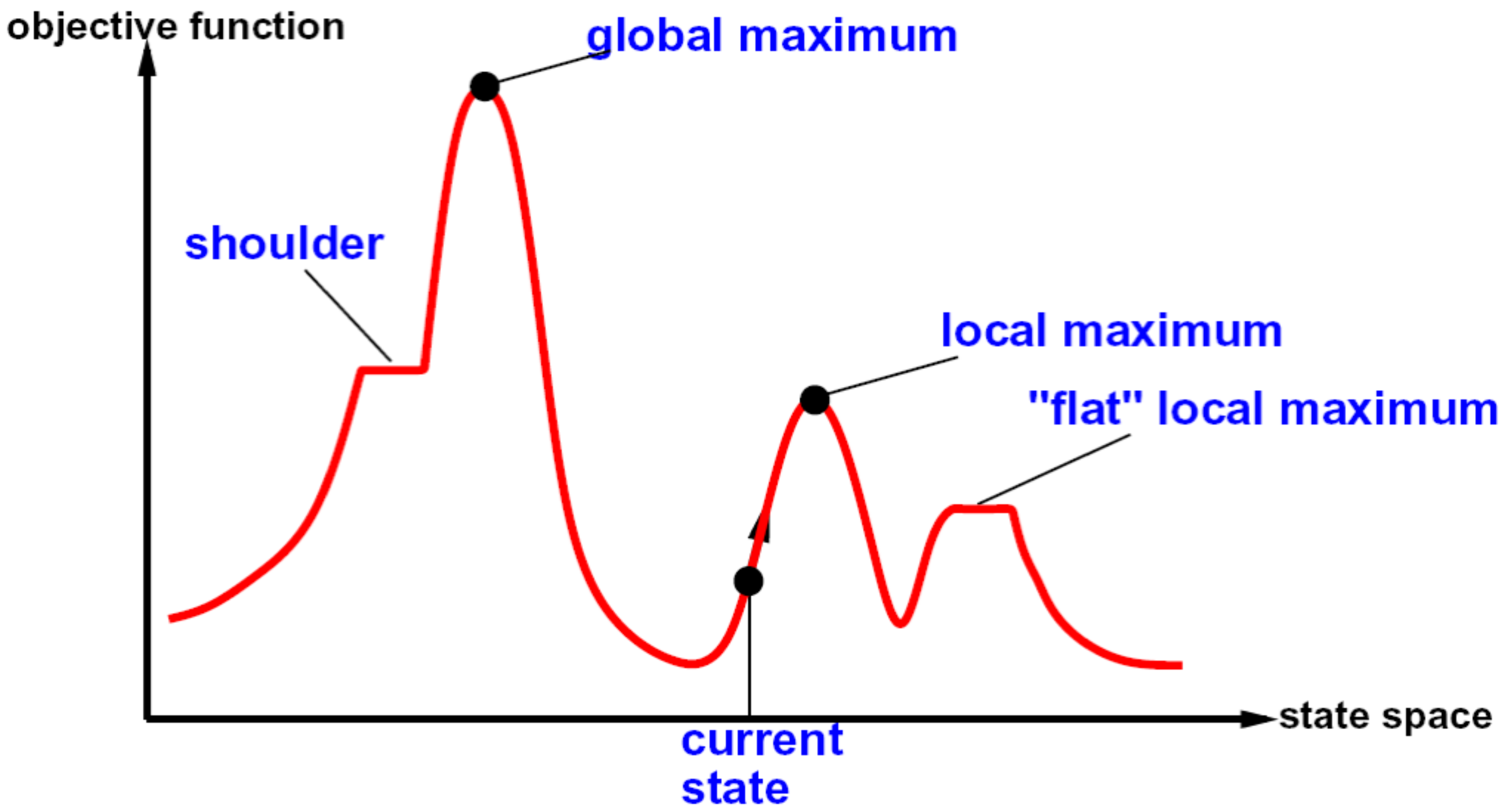
Hill Climbing

- Simple, general idea:
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit

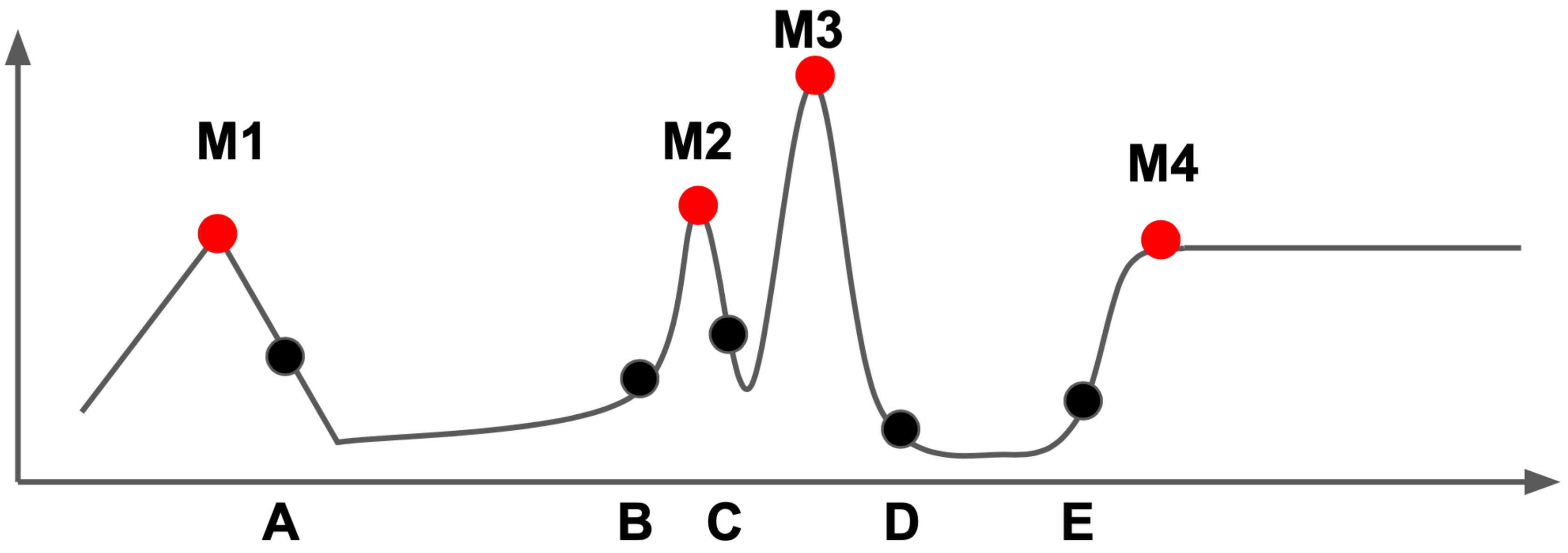
- What's bad about this approach?
  - Complete?
  - Optimal?

- What's good about it?

Hill Climbing Diagram



Exercise: Hill Climbing

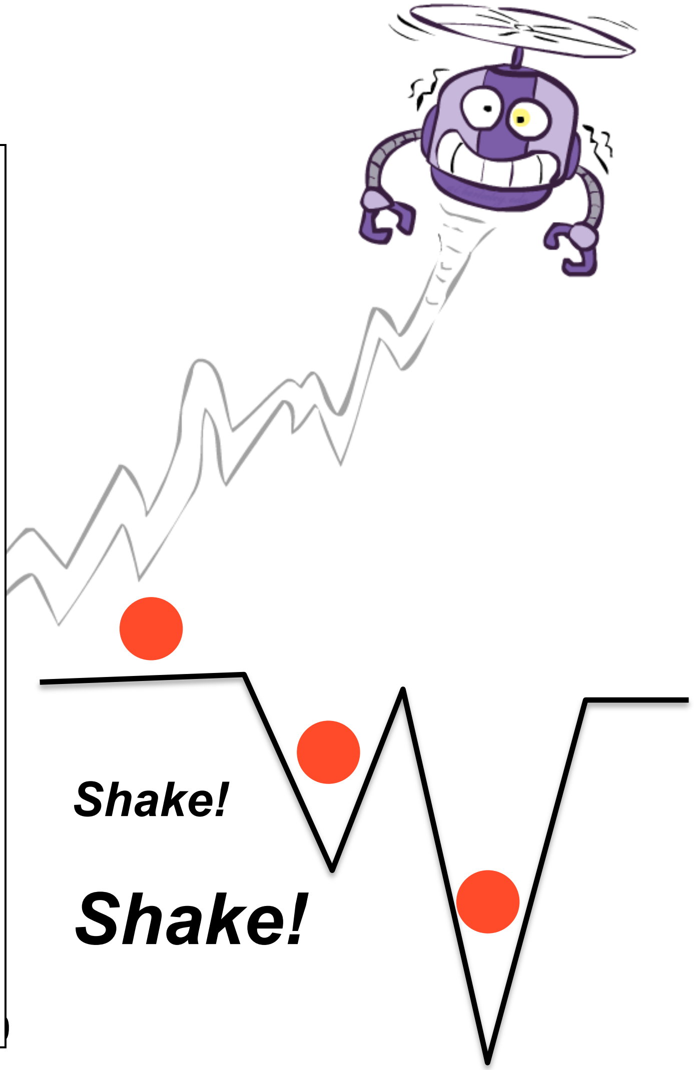


Simulated Annealing

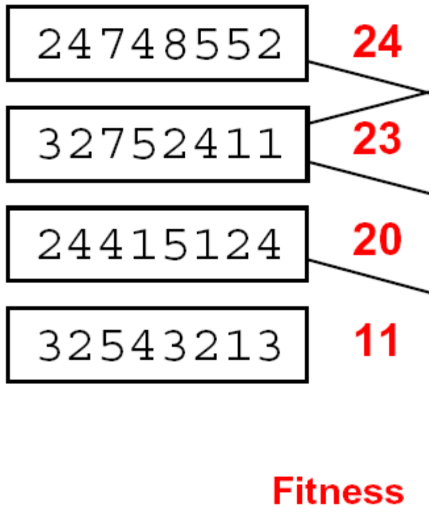
- Idea: Escape local maxima by allowing downhill moves
  - But make them rarer as time goes on

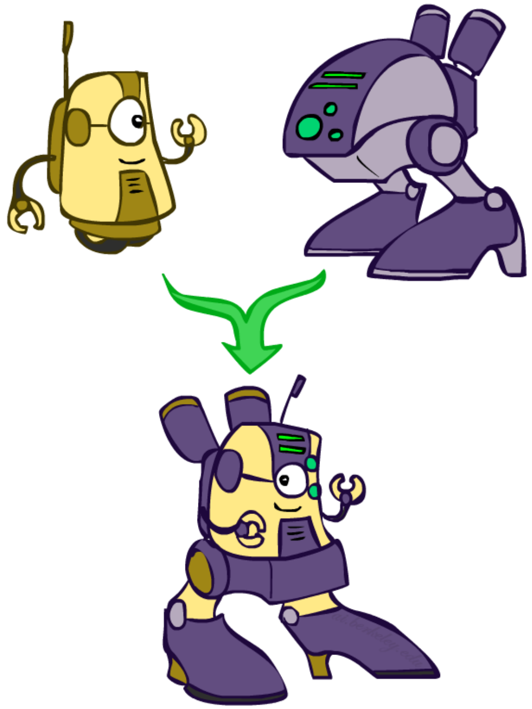
```
function SIMULATED-ANNEALING( problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
       T ← schedule[t]
       if T = 0 then return current
       next ← a randomly selected successor of current
       ΔE ← VALUE[next] – VALUE[current]
       if ΔE > 0 then current ← next
       else current ← next only with probability e^(ΔE/T)
```



Genetic Algorithms

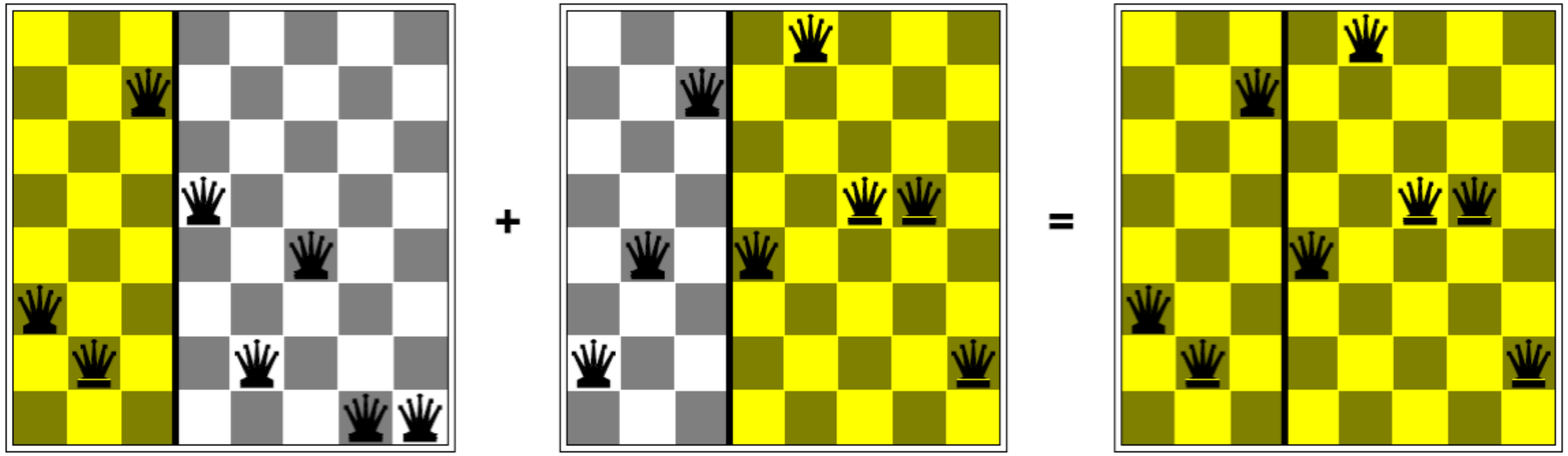




- Genetic algorithms use a natural selection metaphor
  - Keep best N hypotheses at each step (selection) based on a fitness function
  - Also have pairwise crossover operators, with optional mutation to give variety

- Possibly the most misunderstood, misapplied (and even maligned) technique around

Example: N-Queens



- Why does crossover make sense here?
- When wouldn't it make sense?
- What would mutation be?
- What would a good fitness function be?

Gradient Methods

- Continuous state spaces
  - Problem! Cannot select optimal successor

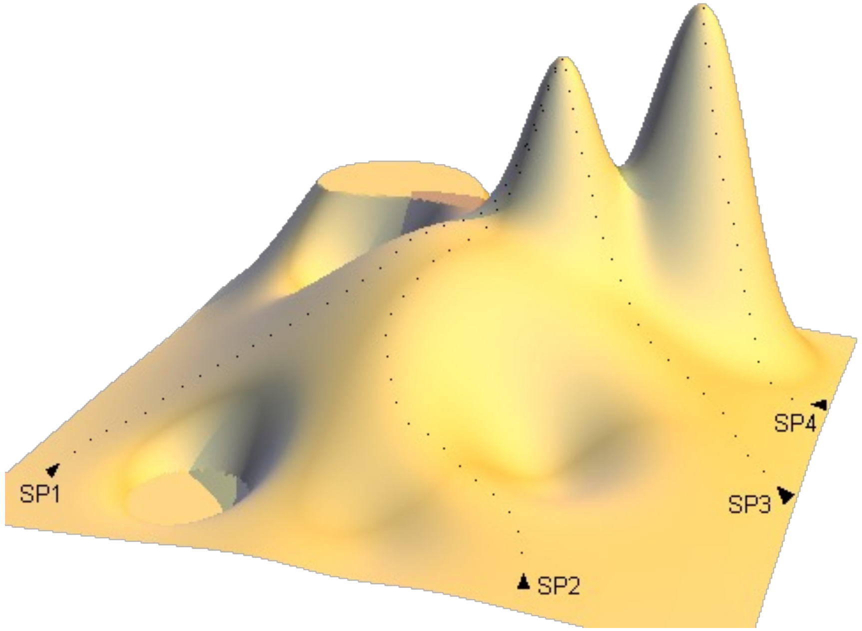
- Discretization or random sampling
  - Choose from a finite number of choices

- Continuous optimization: Gradient ascent
  - Take a step along the gradient (vector of partial derivatives)

- What if you can't compute gradient?
  - i.e. maybe you can only sample the function
  - Estimate gradient from samples!
  - "Stochastic gradient descent"
  - We will return to this in neural networks / deep learning

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3} \right)$$

$$x \leftarrow x + \alpha \nabla f(x)$$



Next Time: Adversarial Search!