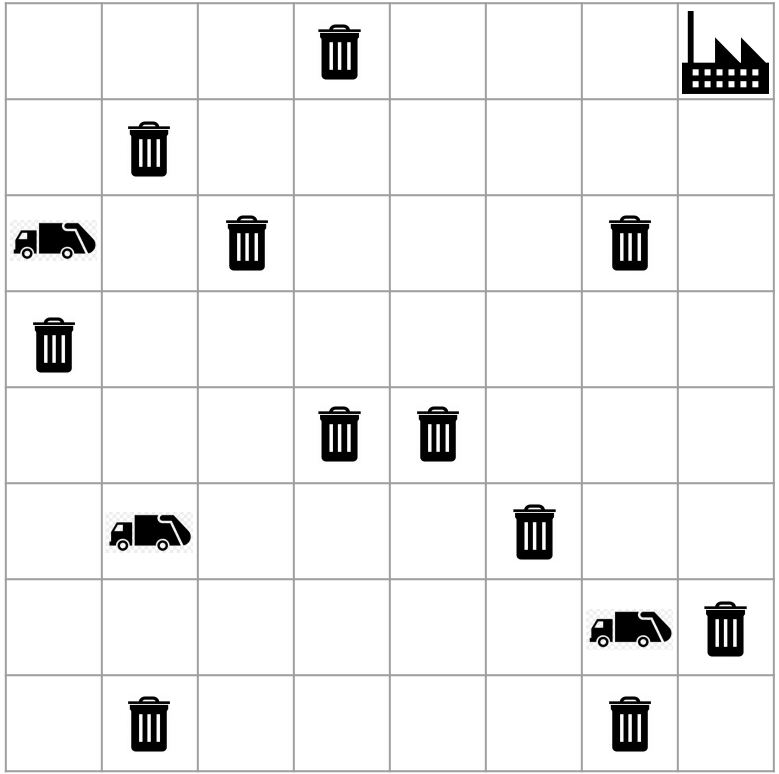
Garbage Collection

In a large maze of size M by N, the garbage collection center controls K garbage trucks to collect the garbage. Each of the trucks starts from its own start location and finally gets back to the city garage g. Along the way, they also need collect all garbage in the maze.

At each step, the center commands all K trucks to move one unit to any adjacent square. The only legal actions are Up, Down, Left, or Right. It is illegal to wait in a square, or to attempt to move into a wall. To collect garbage efficiently, the command center needs to send all trucks to the trash locations and back to the garage in the fewest steps.

Try to come up with a heuristic for this search problem. Is the proposed heuristic admissible?



Potential Heuristics

Some notations:
Manhattan distance between p and q: $MH(p, q)$
The set of positions of remaining garbage: F
The current positions of truck i: p_i

Admissible

1. Number of remaining garbage left divided by K
2. Minimum Manhattan distance between any pair of truck and garbage: $\min_{f \in F}[\min_{1 \le i \le K} MH(f, p_i)]$
3. Maximum Manhattan distance between any pair of truck and the garage: $\max_{1 \le i \le K} MH(p_i, g)$
4. Average of Manhattan distance between all pairs of truck and the garage: $\frac{1}{K} \sum_{k=1}^{K} MH(p_i, g)$
5. Too long to describe in words, so leave the equation here: $\max_{f \in F}[\min_{1 \le i \le K} MH(f, p_i)]$

Inadmissible

6. The average of Manhattan distances between all pairs of truck and garbage
7. Maximum Manhattan distance between any pair of truck and garbage: $\max_{f \in F}[\max_{1 \le i \le K} MH(f, p_i)]$

Explanations of proposed heuristics

Admissible
Heuristic 1 - 3 are trivial.
Heuristic 4 <= heuristic 3.
Heuristic 5 <= the number of steps needed to collect all the garbage

Inadmissible
Heuristic 6, 7: a counterexample is

The average of Manhattan distances between all pairs of truck and garbage is 4.5
The maximum Manhattan distances between all pairs of truck and garbage is 7
But the optimal solution only needs 4 steps.