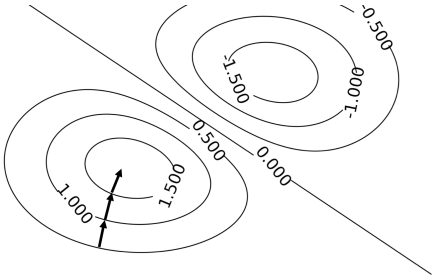
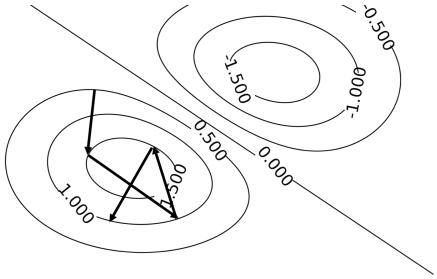
Which of the following paths is a feasible trajectory for the gradient ascent algorithm?

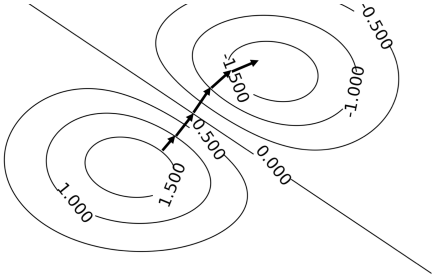
A



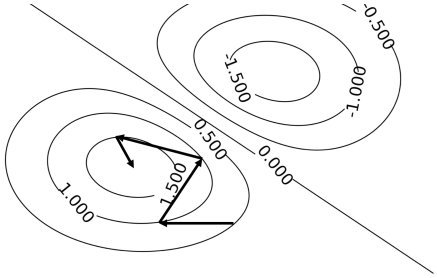
B



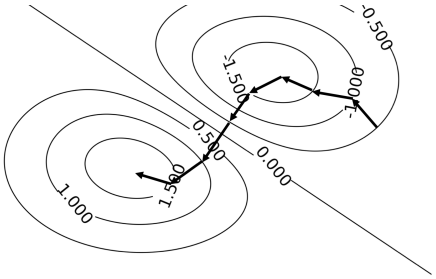
C



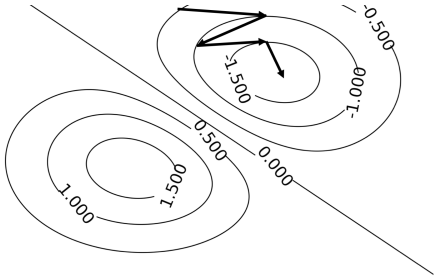
D



E

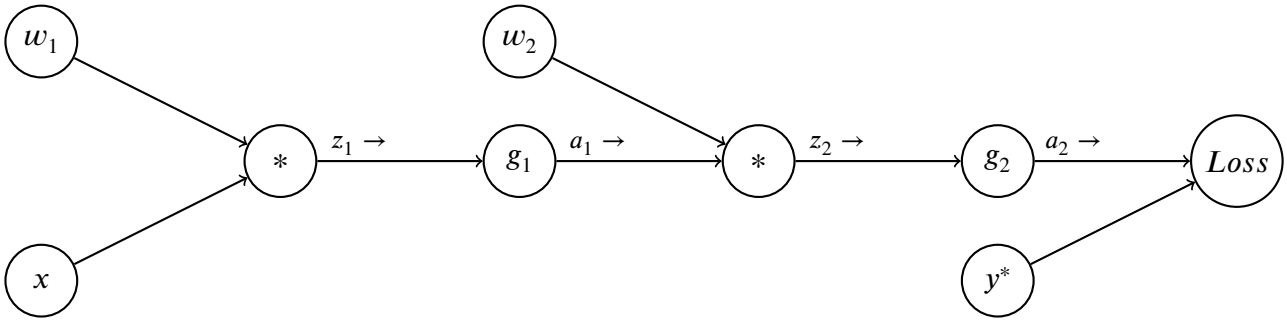


F



Neural Nets

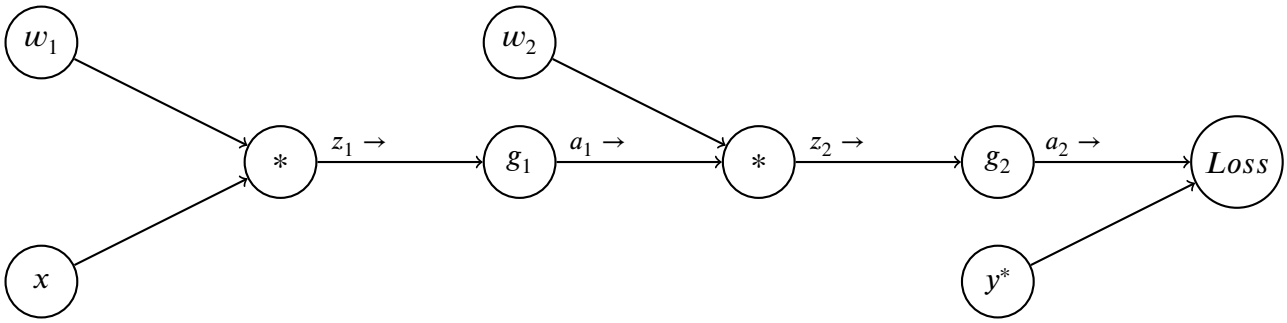
Consider the following computation graph for a simple neural network for binary classification. Here x is a single real-valued input feature with an associated class $y^\star$ (0 or 1). There are two weight parameters w_1 and w_2, and non-linearity functions g_1 and g_2 (to be defined later, below). The network will output a value a_2 between 0 and 1, representing the probability of being in class 1. We will be using a loss function $Loss$ (to be defined later, below), to compare the prediction a_2 with the true class $y^\star$.



1. Perform the forward pass on this network, writing the output values for each node z_1, a_1, z_2 and a_2 in terms of the node's input values:

2. Compute the loss $Loss(a_2, y^\star)$ in terms of the input x, weights w_i, and activation functions g_i:

3. Now we will work through parts of the backward pass, incrementally. Use the chain rule to derive $\frac{\partial Loss}{\partial w_2}$. Write your expression as a product of partial derivatives at each node: i.e. the partial derivative of the node's output with respect to its inputs. (Hint: the series of expressions you wrote in part 1 will be helpful; you may use any of those variables.)



4. Suppose the loss function is quadratic, $Loss(a_2, y^\star) = \frac{1}{2}(a_2 - y^\star)^2$, and g_1 and g_2 are both sigmoid functions $g(z) = \frac{1}{1+e^{-z}}$ (note: it's typically better to use a different type of loss, *cross-entropy*, for classification problems, but we'll use this to make the math easier).

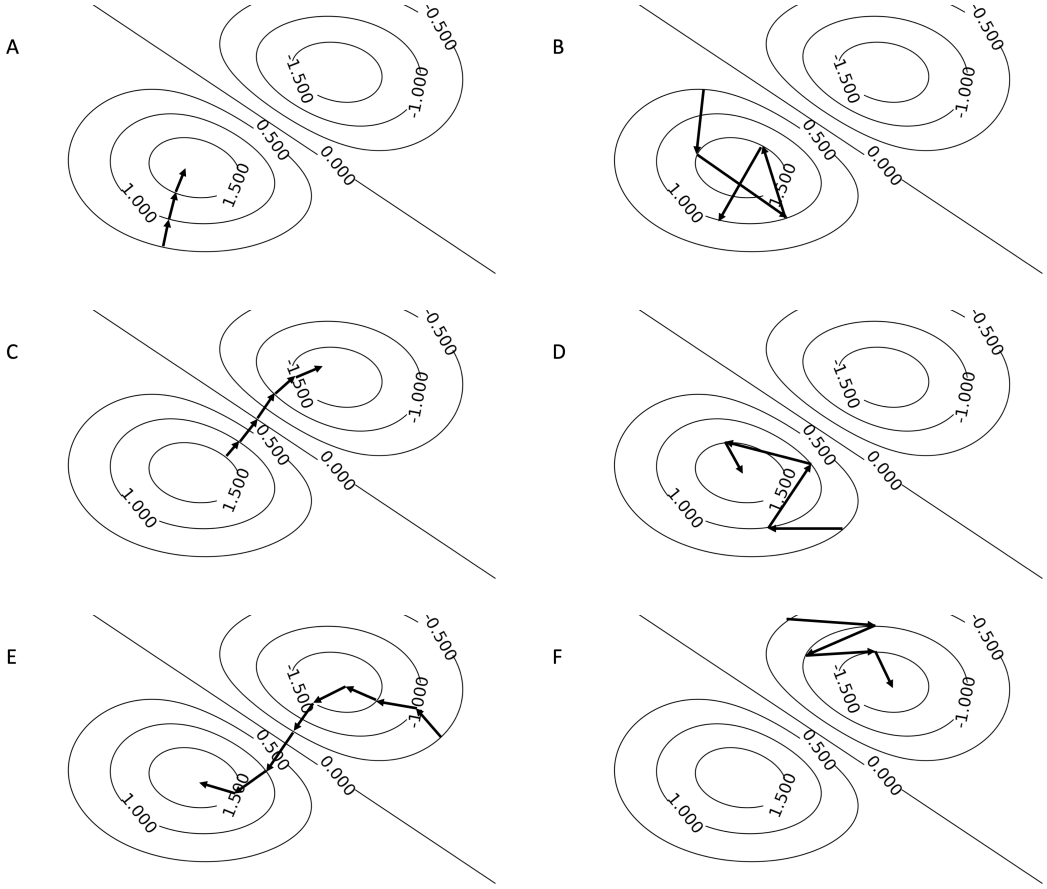
   Using the chain rule from Part 3, and the fact that $\frac{\partial g(z)}{\partial z} = g(z)(1 - g(z))$ for the sigmoid function, write $\frac{\partial Loss}{\partial w_2}$ in terms of the values from the forward pass, $y^\star$, a_1, and a_2:

5. Now use the chain rule to derive $\frac{\partial Loss}{\partial w_1}$ as a product of partial derivatives at each node used in the chain rule:

6. Finally, write $\frac{\partial Loss}{\partial w_1}$ in terms of $x, y^\star, w_i, a_i, z_i$:

7. What is the gradient descent update for w_1 with step-size α in terms of the values computed above?

Which of the following paths is a feasible trajectory for the gradient ascent algorithm?

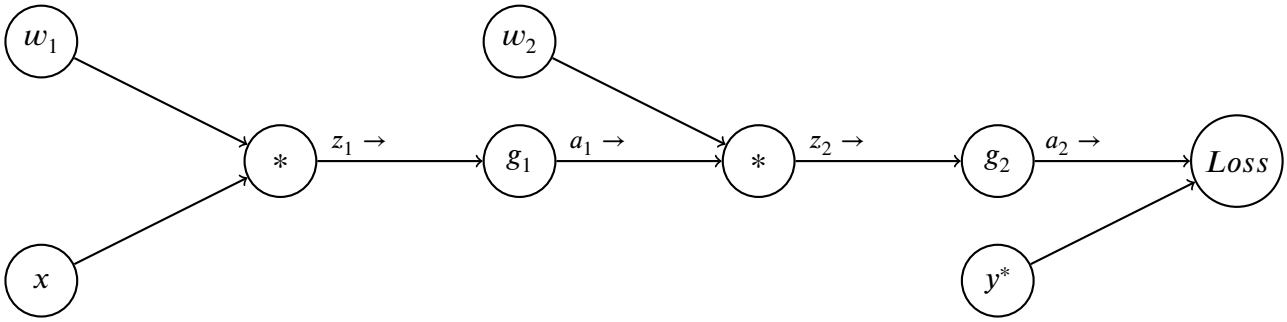


■ A ■ B ☐ C ☐ D ☐ E ☐ F

A is a gradient ascent path since the gradient lines are orthogonal to the contours and the point towards the maximum. B is also a gradient ascent path with a high learning rate. C is not because the path is going towards the minimum instead of the maximum. D is not a gradient ascent path since the gradient is not orthogonal to the contour lines. E is not a gradient ascent path since it starts going towards the minimum. F is not since it goes towards the minimum and the gradients are not orthogonal to the contour lines.

Q2. Neural Nets

Consider the following computation graph for a simple neural network for binary classification. Here x is a single real-valued input feature with an associated class $y^\star$ (0 or 1). There are two weight parameters w_1 and w_2, and non-linearity functions g_1 and g_2 (to be defined later, below). The network will output a value a_2 between 0 and 1, representing the probability of being in class 1. We will be using a loss function $Loss$ (to be defined later, below), to compare the prediction a_2 with the true class $y^\star$.



1. Perform the forward pass on this network, writing the output values for each node z_1, a_1, z_2 and a_2 in terms of the node's input values:

$$z_1 = x * w_1$$
$$a_1 = g_1(z_1)$$
$$z_2 = a_1 * w_2$$
$$a_2 = g_2(z_2)$$

2. Compute the loss $Loss(a_2, y^\star)$ in terms of the input x, weights w_i, and activation functions g_i:

   Recursively substituting the values computed above, we have:

$$Loss(a_2, y^\star) = Loss(g_2(w_2 * g_1(w_1 * x)), y^\star)$$

3. Now we will work through parts of the backward pass, incrementally. Use the chain rule to derive $\frac{\partial Loss}{\partial w_2}$. Write your expression as a product of partial derivatives at each node: i.e. the partial derivative of the node's output with respect to its inputs. (Hint: the series of expressions you wrote in part 1 will be helpful; you may use any of those variables.)

$$\frac{\partial Loss}{\partial w_2} = \frac{\partial Loss}{\partial a_2} \frac{\partial a_2}{\partial z_2} \frac{\partial z_2}{\partial w_2}$$

4. Suppose the loss function is quadratic, $Loss(a_2, y^\star) = \frac{1}{2}(a_2 - y^\star)^2$, and g_1 and g_2 are both sigmoid functions $g(z) = \frac{1}{1+e^{-z}}$ (note: it's typically better to use a different type of loss, *cross-entropy*, for classification problems, but we'll use this to make the math easier).

   Using the chain rule from Part 3, and the fact that $\frac{\partial g(z)}{\partial z} = g(z)(1 - g(z))$ for the sigmoid function, write $\frac{\partial Loss}{\partial w_2}$ in terms of the values from the forward pass, $y^\star$, a_1, and a_2:

   First we'll compute the partial derivatives at each node:

$$\frac{\partial Loss}{\partial a_2} = (a_2 - y^\star)$$
$$\frac{\partial a_2}{\partial z_2} = \frac{\partial g_2(z_2)}{\partial z_2} = g_2(z_2)(1 - g_2(z_2)) = a_2(1 - a_2)$$
$$\frac{\partial z_2}{\partial w_2} = a_1$$

   Now we can plug into the chain rule from part 3:

$$\frac{\partial Loss}{\partial w_2} = \frac{\partial Loss}{\partial a_2}\frac{\partial a_2}{\partial z_2}\frac{\partial z_2}{\partial w_2}$$
$$= (a_2 - y^\star) * a_2(1 - a_2) * a_1$$

5. Now use the chain rule to derive $\frac{\partial Loss}{\partial w_1}$ as a product of partial derivatives at each node used in the chain rule:

$$\frac{\partial Loss}{\partial w_1} = \frac{\partial Loss}{\partial a_2}\frac{\partial a_2}{\partial z_2}\frac{\partial z_2}{\partial a_1}\frac{\partial a_1}{\partial z_1}\frac{\partial z_1}{\partial w_1}$$

6. Finally, write $\frac{\partial Loss}{\partial w_1}$ in terms of $x, y^\star, w_i, a_i, z_i$: The partial derivatives at each node (in addition to the ones we computed in Part 4) are:

$$\frac{\partial z_2}{\partial a_1} = w_2$$
$$\frac{\partial a_1}{\partial z_1} = \frac{\partial g_1(z_1)}{\partial z_1} = g_1(z_1)(1 - g_1(z_1)) = a_1(1 - a_1)$$
$$\frac{\partial z_1}{\partial a_1} = x$$

   Plugging into the chain rule from Part 5 gives:

$$\frac{\partial Loss}{\partial w_1} = \frac{\partial Loss}{\partial a_2}\frac{\partial a_2}{\partial z_2}\frac{\partial z_2}{\partial a_1}\frac{\partial a_1}{\partial z_1}\frac{\partial z_1}{\partial w_1}$$
$$= (a_2 - y^\star) * a_2(1 - a_2) * w_2 * a_1(1 - a_1) * x$$

7. What is the gradient descent update for w_1 with step-size α in terms of the values computed above?

$$w_1 \leftarrow w_1 - \alpha(a_2 - y^\star) * a_2(1 - a_2) * w_2 * a_1(1 - a_1) * x$$