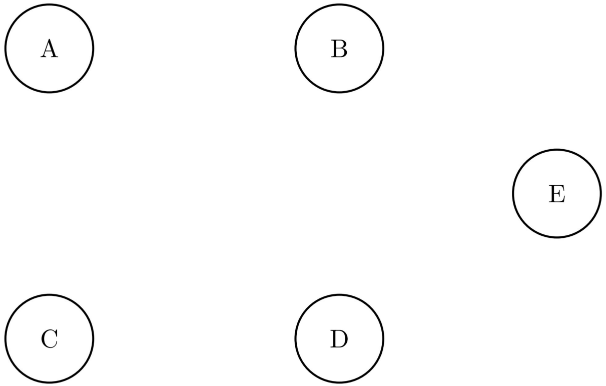
Joint Distributions

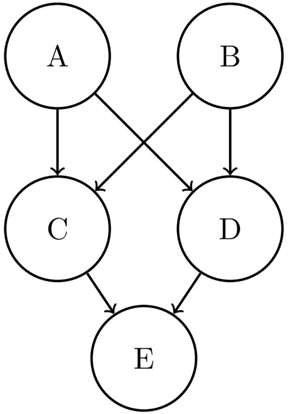
1. Draw the Bayes net associated with the following joint distribution:

   $P(A) \cdot P(B) \cdot P(C|A, B) \cdot P(D|C) \cdot P(E|B, C)$

   Use the right plot as a template.



2. Write down the joint probability distribution associated with the right Bayes Net. Express the answer as a product of terms representing individual conditional probabilities tables.



Joint Distributions

3. Do the following joint probability distributions correspond to a valid Bayes network over the variables A, B, C, D? (Circle TRUE or FALSE.)
   a. TRUE FALSE $P(A) \cdot P(B) \cdot P(C|A) \cdot P(C|B) \cdot P(D|C)$
   b. TRUE FALSE $P(A) \cdot P(B|A) \cdot P(C) \cdot P(D|B, C)$
   c. TRUE FALSE $P(A) \cdot P(B|A) \cdot P(C) \cdot P(C|A) \cdot P(D)$
   d. TRUE FALSE $P(A|B) \cdot P(B|C) \cdot P(C|D) \cdot P(D|A)$

Hint: a specification of Bayes network in Section 14.1 (on the top of Page 511) is attached:

A Bayesian network is a directed graph in which each node is annotated with quantitative probability information. The full specification is as follows:

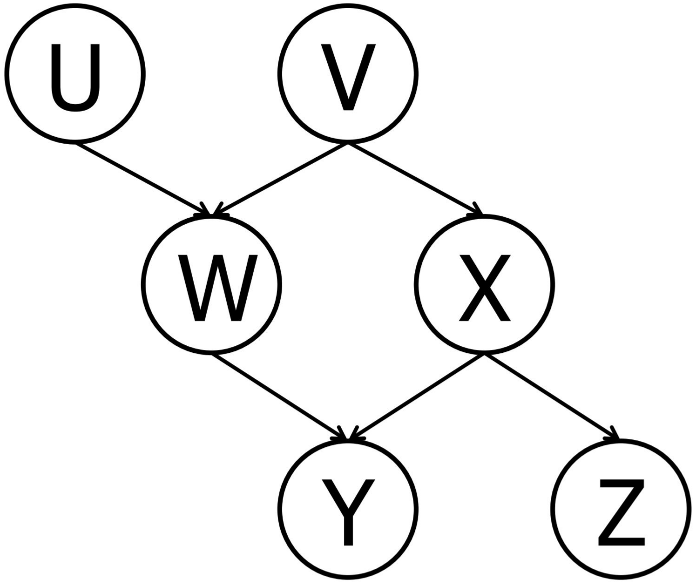
1. Each node corresponds to a random variable, which may be discrete or continuous.
2. A set of directed links or arrows connects pairs of nodes. If there is an arrow from node X to node Y, X is said to be a *parent* of Y. The graph has no directed cycles (and hence is a directed acyclic graph, or DAG.
3. Each node X_i has a conditional probability distribution $\mathbf{P}(X_i \mid Parents(X_i))$ that quantifies the effect of the parents on the node.

Independence

Based only on the structure of the Bayes' Net given right, indicate whether the following conditional independence assertions are either 1) *guaranteed to be true*, 2) *guaranteed to be false*, or 3) *cannot be determined* by the structure alone.
Hint:

- the meaning of $A \perp B \mid C, D$ is A and B are independent ($\perp$) of each other conditioned on (|) C and D
- Two properties about independence are in Fig 14.4 (Page 518) in the textbook.

i. $U \perp V$
ii. $U \perp V \mid W$
iii. $U \perp Z \mid W$
iv. $U \perp Z \mid X, W$
v. $V \perp Z \mid X$



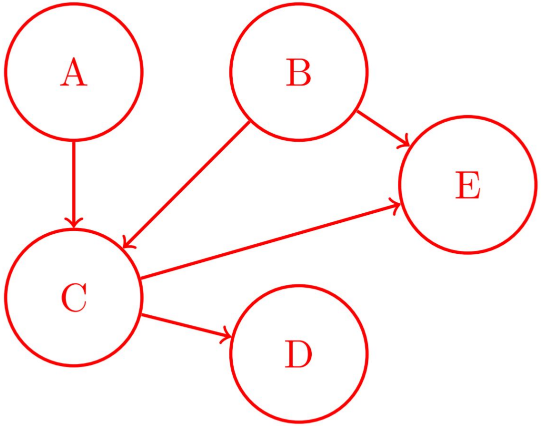
Joint Distributions Answers

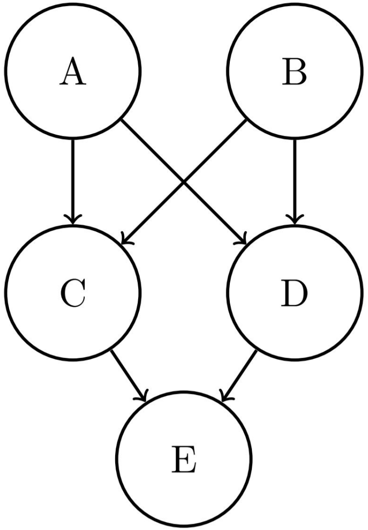
1. Draw the Bayes net associated with the following joint distribution:

   $P(A) \cdot P(B) \cdot P(C|A, B) \cdot P(D|C) \cdot P(E|B, C)$

2. Use the right plot as a template.Write down the joint probability distribution associated with the right Bayes Net. Express the answer as a product of terms representing individual conditional probabilities tables.

   $P(A) \cdot P(B) \cdot P(C|A, B) \cdot P(D|A, B) \cdot P(E|C, D)$





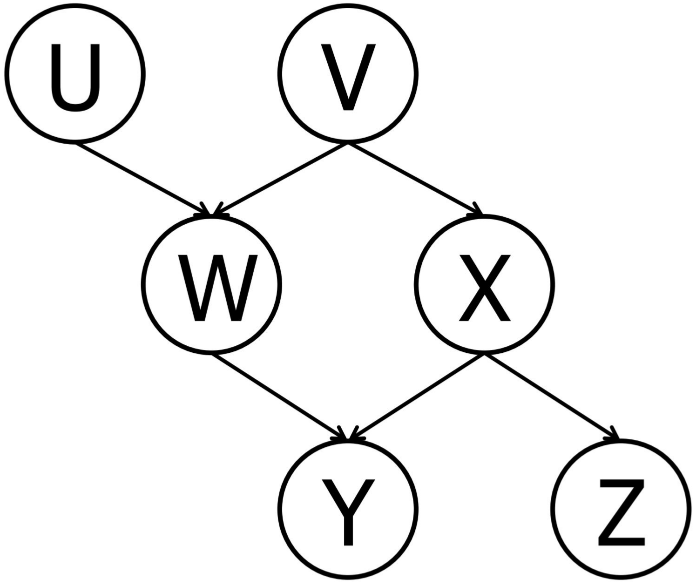
Joint Distributions

3. Do the following joint probability distributions correspond to a valid Bayes network over the variables A, B, C, D? (Circle TRUE or FALSE.)
   a. TRUE FALSE $P(A) \cdot P(B) \cdot P(C|A) \cdot P(C|B) \cdot P(D|C)$
      C must be either conditioned on 1) A only, 2) B only, or 3) both A and B
   b. TRUE FALSE $P(A) \cdot P(B|A) \cdot P(C) \cdot P(D|B, C)$
   c. TRUE FALSE $P(A) \cdot P(B|A) \cdot P(C) \cdot P(C|A) \cdot P(D)$
      The same reason as in question a, and P(C) also indicates C is independent of all other variables and thus it's a further contradiction
   d. TRUE FALSE $P(A|B) \cdot P(B|C) \cdot P(C|D) \cdot P(D|A)$
      There is a cycle of dependence.

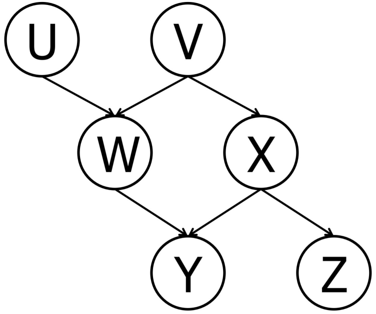
Independence

Based only on the structure of the Bayes' Net given right, indicate whether the following conditional independence assertions are either 1) *guaranteed to be true*, 2) *guaranteed to be false*, or 3) *cannot be determined* by the structure alone.

Hint: the meaning of A ⊥ B | C, D is A and B are independent (⊥) of each other other conditioned on (|) C and D



i. U ⊥ V — guaranteed to be true
ii. U ⊥ V | W — cannot be determined
iii. U ⊥ Z | W — cannot be determined
iv. U ⊥ Z | X, W — guaranteed to be true
v. V ⊥ Z | X — guaranteed to be true



Independence Answers

i. U ⊥ V guaranteed to be true

obvious from the net

ii. U ⊥ V | W cannot be determined

When measuring a common effect of two independent causes, the causes usually becomes dependent, because finding the truth of one makes the other less likely (or "explained away"), and refuting one implies the truth of the other. But we cannot choose “guaranteed to be false” neither because they can be independent. A conditionally dependent example is shown below. Try to come up with a conditionally independent example yourself:

$$P(U=T|W=T) = \frac{\sum_{v\in\{T,F\}}[P(W=T|U=T,V=v)P(U=T)P(V=v)]}{\sum_{u\in\{T,F\}}\left[\sum_{v\in\{T,F\}}[P(W=T|U=u,V=v)P(U=u)P(V=v)]\right]}$$

$$= \frac{1.0\cdot 0.5\cdot 0.5 + 0.9\cdot 0.5\cdot 0.5}{1.0\cdot 0.5\cdot 0.5 + 0.9\cdot 0.5\cdot 0.5 + 0.9\cdot 0.5\cdot 0.5 + 0.0\cdot 0.5\cdot 0.5}$$

$$= \frac{19}{28}$$

$$P(V=T|W=T) = \frac{19}{28}\text{(Calculation similar to above)}$$

$$P(U=T,V=T|W=T) = \frac{P(W=T|U=T,V=T)P(U=T)P(V=T)}{\sum_{u\in\{T,F\}}\left[\sum_{v\in\{T,F\}}[P(W=T|U=u,V=v)P(U=u)P(V=v)]\right]}$$

$$= \frac{1.0\cdot 0.5\cdot 0.5}{1.0\cdot 0.5\cdot 0.5 + 0.9\cdot 0.5\cdot 0.5 + 0.9\cdot 0.5\cdot 0.5 + 0.0\cdot 0.5\cdot 0.5}$$

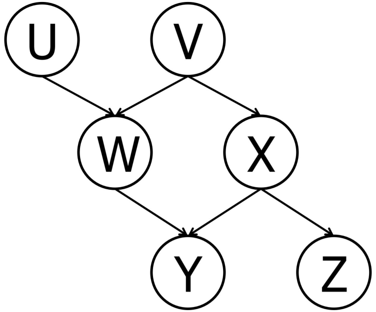
$$= \frac{10}{28}$$

Obviously, $P(U=T,V=T|W=T) \neq P(U=T|W=T)\cdot P(V=T|W=T)$

P(U=T)	P(U=T)	P(V=T)	P(V=T)
0.5	0.5	0.5	0.5

U	W	P(W=T | U, V)	P(W=F | U, V)
T	T	1.0	0.0
T	F	0.9	0.1
F	T	0.1	0.9
F	F	0.0	1.0

Independence Answers



iii. $U \perp Z \mid W$ cannot be determined

For the same reason in 2, V can be conditionally dependent of U given W, where V further affects Z, so V and Z are conditionally dependent given W

iv. $U \perp Z \mid X, W$ guaranteed to be true

The probability distribution of Z can be completely determined from the given X, so it's independent of U then.

v. $V \perp Z \mid X$ guaranteed to be true

Similar reason as 5