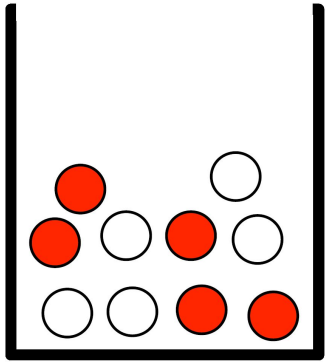
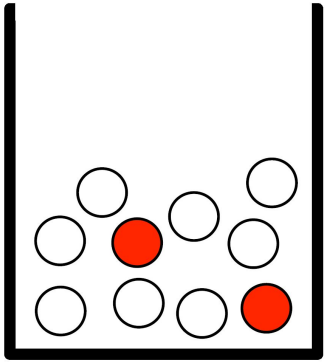
After a tiring day of eating food and escaping from ghosts, Pacman heads to the casino for some well-deserved rest and relaxation! This particular casino has two games, Low and High, which are both free to play.

The two games are set up very similarly. In each game, there is a bin of marbles. The Low bin contains 5 white and 5 dark marbles, and the High bin contains 8 white and 2 dark marbles.

Play for each game proceeds as follows: the dealer draws a single marble at random from the bin. If a dark marble is drawn, the game pays out. The Low payout is $100, and the High payout is $1000. The payout is **divided evenly** among everyone playing that game. For example, if two people are playing Low and a dark marble is drawn, they each receive $50. If a white marble is drawn, they receive nothing. The drawings for both games are done simultaneously, and only once per night (there is no repeated play).



Low: $100



High: $1000

(a) **Expectations**. Suppose Pacman is at the casino by himself (there are no other players). Give his expected winnings, in dollars:

(i) From playing a single round of Low

(ii) From playing a single round of High

(b) **Preferences**. Pacman is still at the casino by himself. Let p denote the amount of money Pacman wins, and let his utility be given by some function $U(p)$. Assume that Pacman is a rational agent who acts to maximize expected utility.

(i) If you observe that Pacman chooses to play Low, which of the following must be true about $U(p)$? Assume $U(0) = 0$. (choose any that apply)

$U(50) \geq U(1000)$ $U(100) \geq U(1000)$ $\frac{1}{2}\, U(100) \geq \frac{1}{5}\, U(1000)$ $U(50) \geq U(100)$

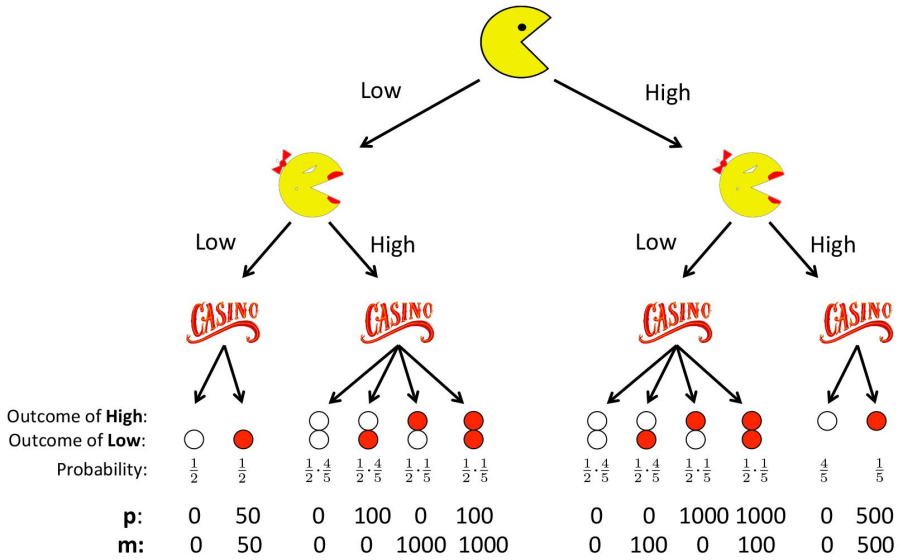
(ii) Given that Pacman plays Low, which of the following are possibilities for $U(p)$? (choose any that apply, for reference, $\sqrt[3]{100} \approx 4.6$)

p $-p$ $2^p - 1$ p^2 $\sqrt[3]{p}$

(c) **Multiple Players**. Ms. Pacman is joining Pacman at the casino! Assume that Pacman arrives first and chooses which game he will play, and then Ms. Pacman arrives and chooses which game she will play. Let *p* denote Pacman's winnings and *m* denote Ms. Pacman's winnings. Since both Pacman and Ms. Pacman are rational agents, we can describe Pacman's utility with a function $U_1(p, m)$ and Ms. Pacman's utility with a function $U_2(p, m)$. You might find it helpful to refer to the game tree given on the right.

(i) Suppose $U_1(p, m) = p$ and $U_2(p, m) = m$; that is, both players are attempting to maximize their own expected winnings. Compute the expected utilities of both players, for each combination of games they could play:

Pacman	Ms. Pacman	$\mathbb{E}[U_1(p,m)]$	$\mathbb{E}[U_2(p,m)]$
Low	Low		
Low	High		
High	Low		
High	High		



(ii) Given that Pacman chooses first, which of the following are possibilities for the games Pacman and Ms. Pacman respectively choose to play? (circle all that apply)

(Low, Low) (Low, High) (High, Low) (High, High)

(iii) **Scenarios**. Now rather than simply maximizing their own winnings, Pacman and Ms. Pacman have different objectives. Here are five utility functions $U_1(p, m)$ for Pacman:

$$p \qquad p+m \qquad m \qquad (p+m)^2 \qquad -m$$

and five utility functions $U_2(p, m)$ for Ms. Pacman:

$$m \qquad p+m \qquad -p \qquad 2m-p \qquad \log_{10} m$$

For each of the following scenarios, give the utility functions listed above which best encode the motivations of each player. A particular function may appear more than once. The first scenario is done for you.

Senario	**Pacman**	**Ms.Pacman**
Pacman and Ms. Pacman each want to maximize their own expected winnings.	p	m
Pacman and Ms. Pacman have had a terrible fight and are very angry at each other. Each wants the other to lose as much money as possible.		
Pacman has gotten over the fight, and now wants to maximize their expected combined winnings (since Pacman and Ms. Pacman share a bank account). However, Ms. Pacman does not trust Pacman to deposit his share, so she just wants to maximize her own expected winnings.		
Pacman is being extorted by the Ghost Mafia, who will immediately confiscate any money that he wins (that is, if Pacman wins$100, he will still have p = 100 but does not actually get to keep the money). The Mafia is not monitoring Ms. Pacman and does not know about her winnings, so they will not be confiscated. Both Pacman and Ms. Pacman want to maximize the expected total amount the couple gets to keep.		

(a) **Expectations**. Suppose Pacman is at the casino by himself (there are no other players). Give his expected winnings, in dollars:

(i) From playing a single round of Low: 5/10 * $100 = $50

(ii) From playing a single round of High: 2/10 * $1000 = $200

(b) **Preferences**. Pacman is still at the casino by himself. Let p denote the amount of money Pacman wins, and let his utility be given by some function $U(p)$. Assume that Pacman is a rational agent who acts to maximize expected utility.

(i) If you observe that Pacman chooses to play Low, which of the following must be true about $U(p)$? Assume $U(0) = 0$. (choose any that apply)

$U(50) \geq U(1000)$ $U(100) \geq U(1000)$ $\frac{1}{2}\, U(100) \geq \frac{1}{5}\, U(1000)$ $U(50) \geq U(100)$

(ii) Given that Pacman plays Low, which of the following are possibilities for $U(p)$? (choose any that apply)

p $-p$ $2^p - 1$ p^2 $\sqrt[3]{p}$

(iii) **Scenarios**. Now rather than simply maximizing their own winnings, Pacman and Ms. Pacman have different objectives. Here are five utility functions $U_1(p, m)$ for Pacman:

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$$m \qquad p+m \qquad -p \qquad 2m-p \qquad \log_{10} m$$

For each of the following scenarios, give the utility functions listed above which best encode the motivations of each player. A particular function may appear more than once. The first scenario is done for you.

Senario	**Pacman**	**Ms.Pacman**
Pacman and Ms. Pacman each want to maximize their own expected winnings.	p	m
Pacman and Ms. Pacman have had a terrible fight and are very angry at each other. Each wants the other to lose as much money as possible.	$-m$	$-p$
Pacman has gotten over the fight, and now wants to maximize their expected combined winnings (since Pacman and Ms. Pacman share a bank account). However, Ms. Pacman does not trust Pacman to deposit his share, so she just wants to maximize her own expected winnings.	$p+m$	m
Pacman is being extorted by the Ghost Mafia, who will immediately confiscate any money that he wins (that is, if Pacman wins$100, he will still have p = 100 but does not actually get to keep the money). The Mafia is not monitoring Ms. Pacman and does not know about her winnings, so they will not be confiscated. Both Pacman and Ms. Pacman want to maximize the expected total amount the couple gets to keep.	m	m