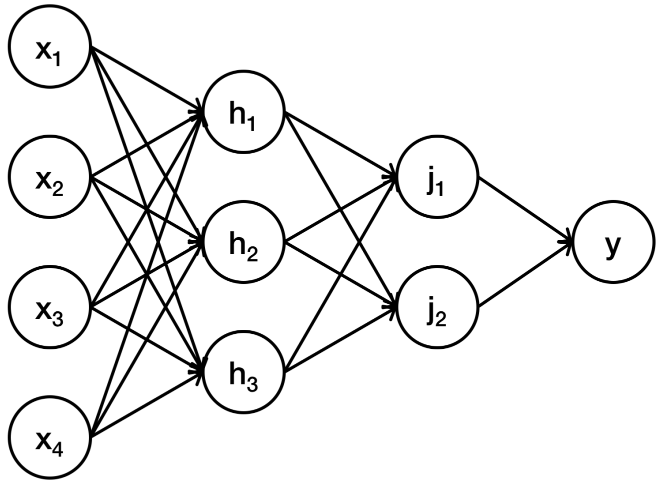
Neural Network Data Sufficiency

The next few problems use the below neural network as a reference. Neurons h_{1-3} and j_{1-2} all use ReLU activation functions. Neuron y uses the identity activation function: $f(x) = x$. In the questions below, let $w_{a,b}$ denote the weight that connects neurons a and b. Also, let o_a denote the value that neuron a outputs to its next layer.



Given this network, in the following few problems, you have to decide whether the data given are sufficient for answering the question.

(a) [2 pts] Given the above neural network, what is the value of o_y?

Data item 1: the values of all weights in the network and the values o_{h_1}, o_{h_2}, o_{h_3}
Data item 2: the values of all weights in the network and the values o_{j_1}, o_{j_2}

- ◯ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ◯ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
- ◯ Both statements taken together are sufficient, but neither data item alone is sufficient.
- ◯ Each data item alone is sufficient to answer the question.
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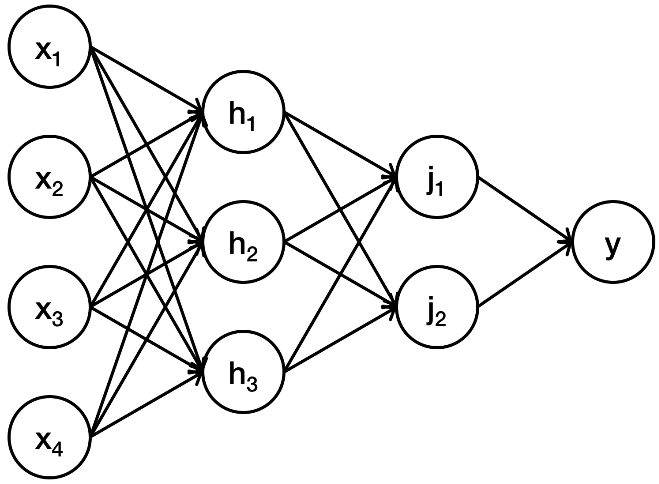

Given this network, in the following few problems, you have to decide whether the data given are sufficient for answering the question.

(b) [2 pts] Given the above neural network, what is the value of o_{h_1}?

Data item 1: the neuron input values, i.e., o_{x_1} through o_{x_4}
Data item 2: the values o_{j_1}, o_{j_2}

- ◯ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
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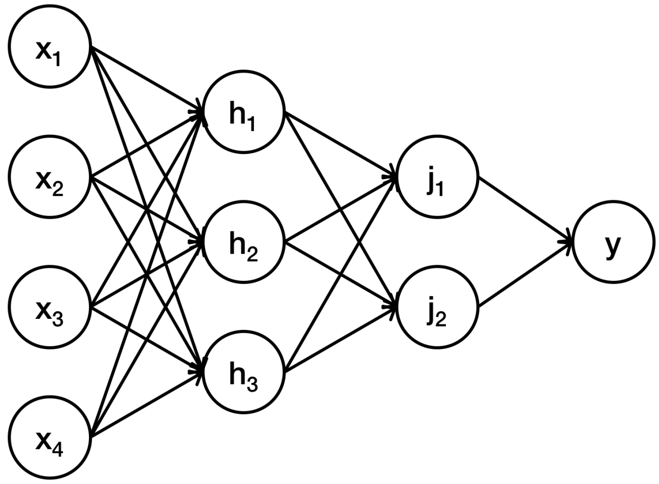

Given this network, in the following few problems, you have to decide whether the data given are sufficient for answering the question.

(c) [2 pts] Given the above neural network, what is the value of o_{j_1}?

Data item 1: the values of all weights connecting neurons h_1, h_2, h_3 to j_1, j_2
Data item 2: the values o_{h_1}, o_{h_2}, o_{h_3}

- ◯ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
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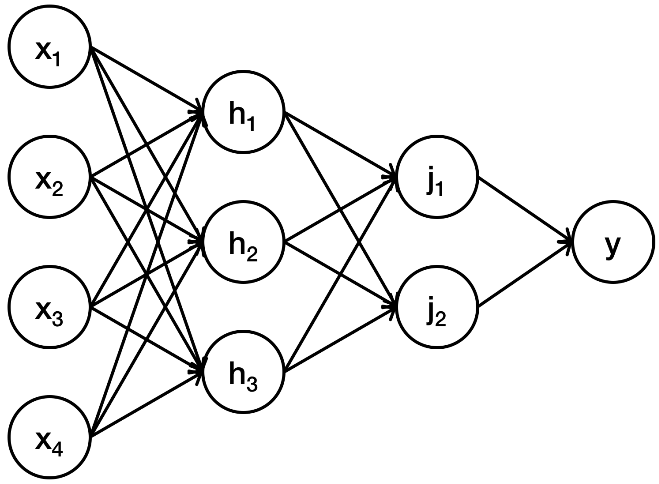

Given this network, in the following few problems, you have to decide whether the data given are sufficient for answering the question.

(d) [2 pts] Given the above neural network, what is the value of $\partial o_y / \partial w_{j_2,y}$?

Data item 1: the value of o_{j_2}
Data item 2: all weights in the network and the neuron input values, i.e., o_{x_1} through o_{x_4}

◯ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
◯ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
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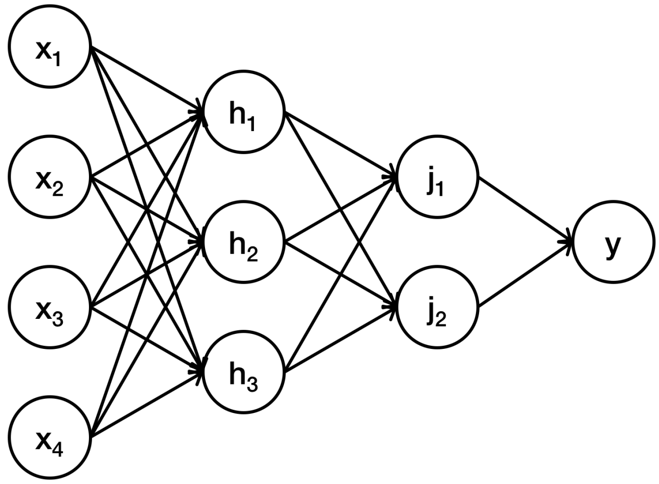

Given this network, in the following few problems, you have to decide whether the data given are sufficient for answering the question.

(e) [2 pts] Given the above neural network, what is the value of $\partial o_y / \partial w_{h_2,j_2}$?

Data item 1: the value of $w_{j_2,y}$
Data item 2: the value of $\partial o_{j_2} / \partial w_{h_2,j_2}$

○ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
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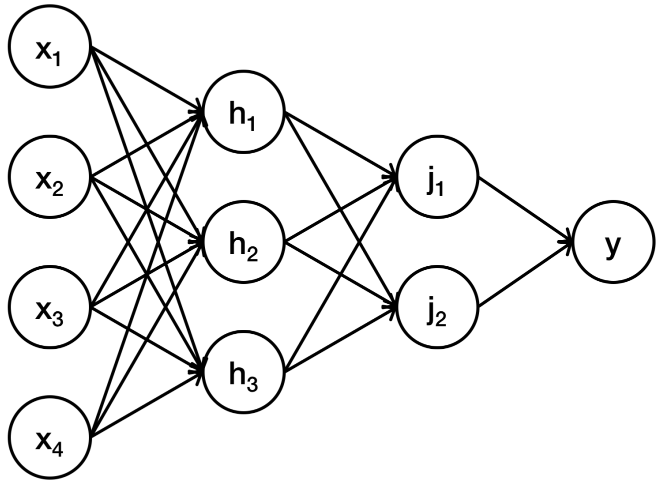

Given this network, in the following few problems, you have to decide whether the data given are sufficient for answering the question.

(f) [2 pts] Given the above neural network, what is the value of $\partial o_y / \partial w_{x_1,h_3}$?

Data item 1: the value of all weights in the network and the neuron input values, i.e., o_{x_1} through o_{x_4}
Data item 2: the value of w_{x_1,h_3}

- ◯ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ◯ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
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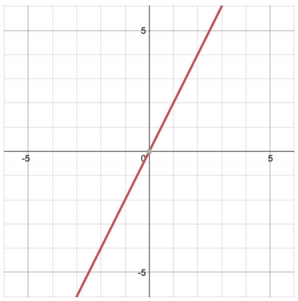
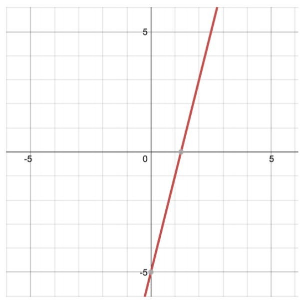
Neural Network Representations

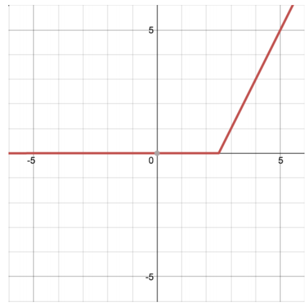
You are given a number of functions (a-h) of a single variable, x, which are graphed below. The computation graphs on the following pages will start off simple and get more complex, building up to neural networks. For each computation graph, indicate which of the functions below they are able to represent.



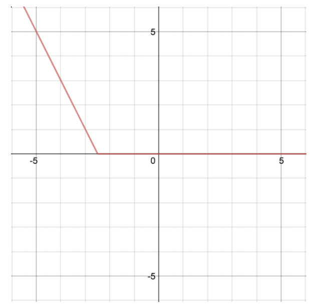
(a) $2x$



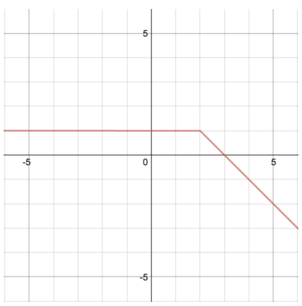
(b) $4x - 5$



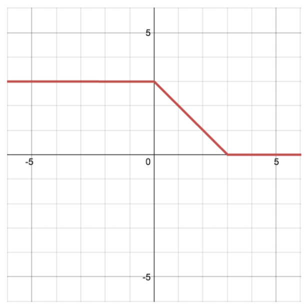
(c) $\begin{cases} 2x - 5 & x \geq 2.5 \\ 0 & x < 2.5 \end{cases}$



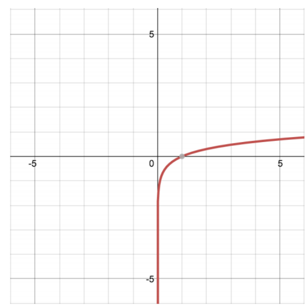
(d) $\begin{cases} -2x - 5 & x \leq -2.5 \\ 0 & x > -2.5 \end{cases}$



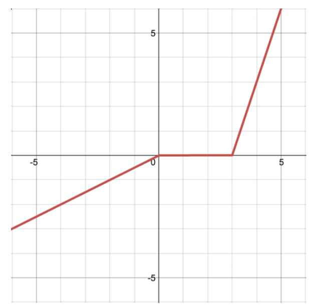
(e) $\begin{cases} -x + 3 & x \geq 2 \\ 1 & x < 2 \end{cases}$



(f) $\begin{cases} 3 & x \leq 0 \\ 3 - x & 0 < x \leq 3 \\ 0 & x > 3 \end{cases}$

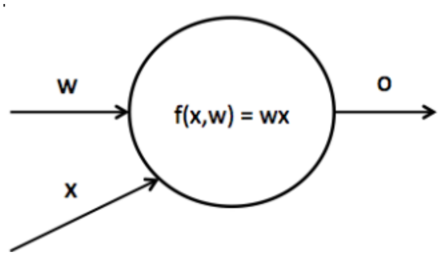


(g) $\log(x)$



(h) $\begin{cases} 0.5x & x \leq 0 \\ 0 & 0 < x \leq 3 \\ 3x - 9 & x > 3 \end{cases}$

1. Consider the following computation graph, computing a linear transformation with scalar input x, weight w, and output o, such that $o = wx$. Which of the funcions can be represented by this graph? For the options which can, write out the appropriate value of w.



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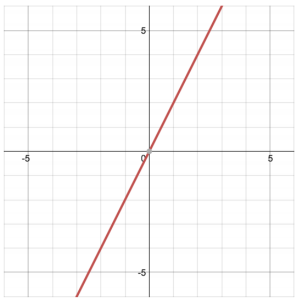
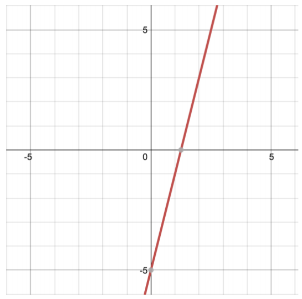
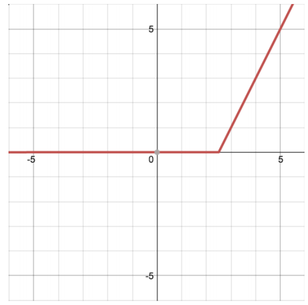


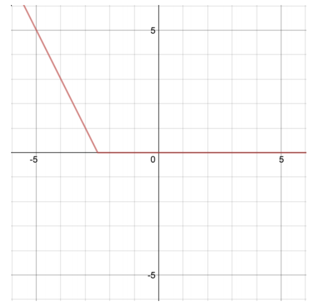
(a) $2x$



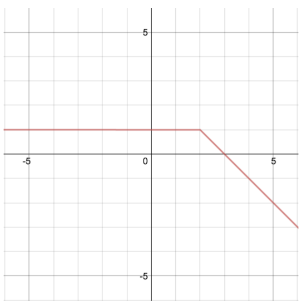
(b) $4x - 5$



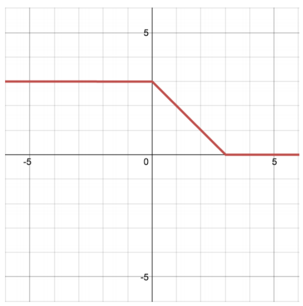
(c) $\begin{cases} 2x - 5 & x \geq 2.5 \\ 0 & x < 2.5 \end{cases}$



(d) $\begin{cases} -2x - 5 & x \leq -2.5 \\ 0 & x > -2.5 \end{cases}$



(e) $\begin{cases} -x + 3 & x \geq 2 \\ 1 & x < 2 \end{cases}$



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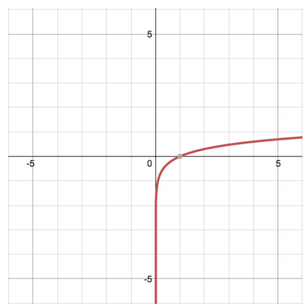
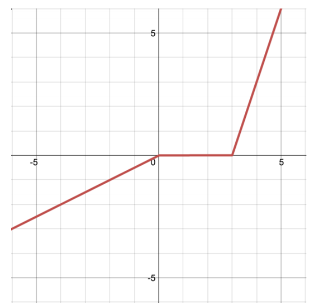
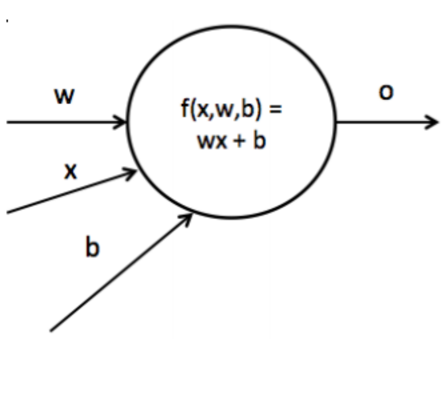


(g) $\log(x)$



(h) $\begin{cases} 0.5x & x \leq 0 \\ 0 & 0 < x \leq 3 \\ 3x - 9 & x > 3 \end{cases}$

2. Now we introduce a bias term b into the graph, such that $o = wx + b$ (this is known as an *affine* function). Which of the functions can be represented by this network? For the options which can, write out an appropriate value of w, b.



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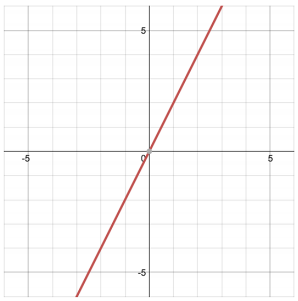
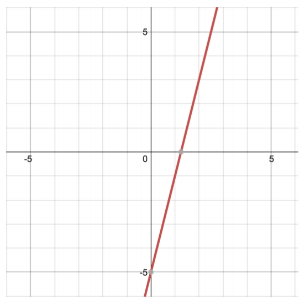
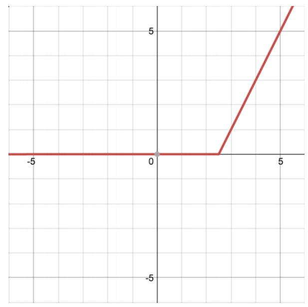


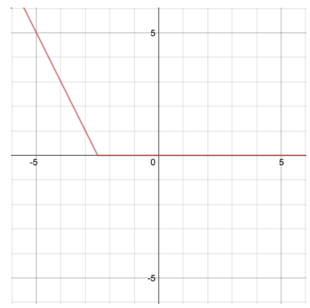
(a) $2x$



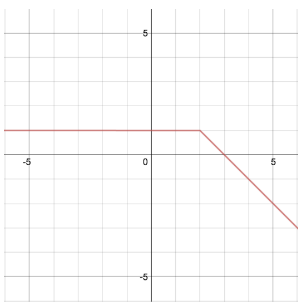
(b) $4x - 5$



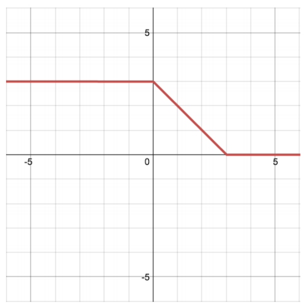
(c) $\begin{cases} 2x - 5 & x \geq 2.5 \\ 0 & x < 2.5 \end{cases}$



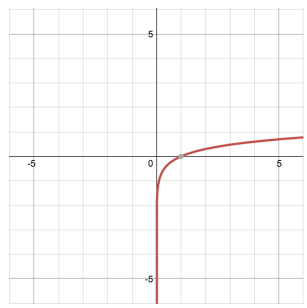
(d) $\begin{cases} -2x - 5 & x \leq -2.5 \\ 0 & x > -2.5 \end{cases}$



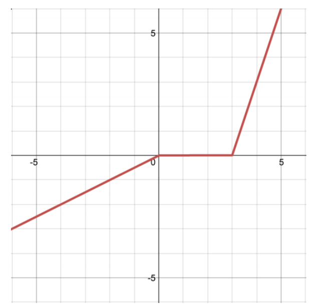
(e) $\begin{cases} -x + 3 & x \geq 2 \\ 1 & x < 2 \end{cases}$



(f) $\begin{cases} 3 & x \leq 0 \\ 3 - x & 0 < x \leq 3 \\ 0 & x > 3 \end{cases}$

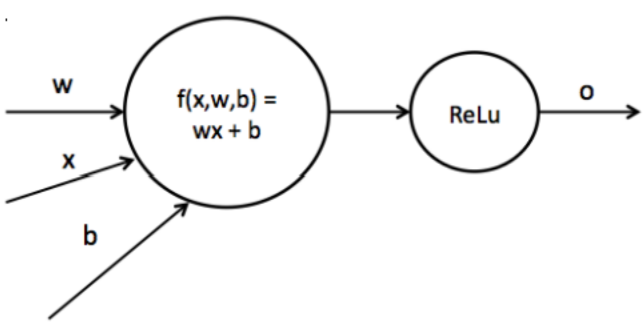


(g) $\log(x)$



(h) $\begin{cases} 0.5x & x \leq 0 \\ 0 & 0 < x \leq 3 \\ 3x - 9 & x > 3 \end{cases}$

3. We can introduce a non-linearity into the network as indicated below. We use the ReLU non-linearity, which has the form $ReLU(x) = \max(0, x)$. Now which of the functions can be represented by this neural network with weight w and bias b? For the options which can, write out an appropriate value of w, b.



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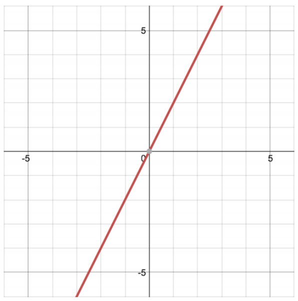
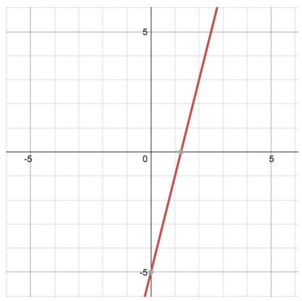
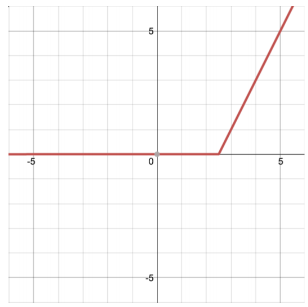


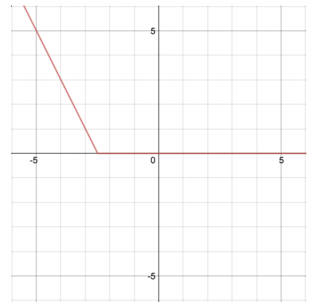
(a) $2x$



(b) $4x - 5$



(c) $\begin{cases} 2x-5 & x \geq 2.5 \\ 0 & x < 2.5 \end{cases}$



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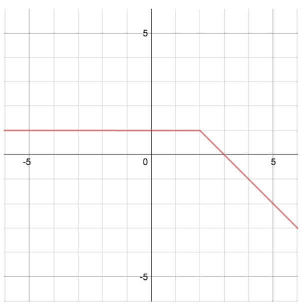
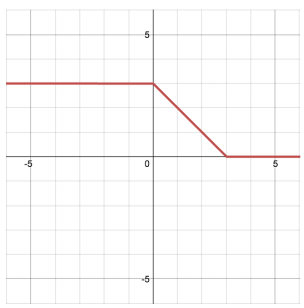
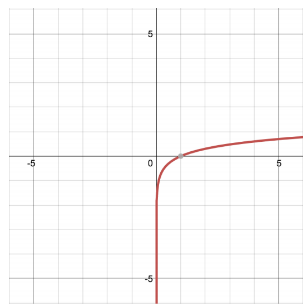


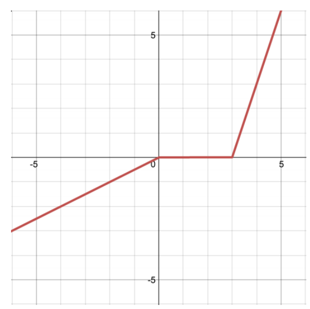
(e) $\begin{cases} -x+3 & x \geq 2 \\ 1 & x < 2 \end{cases}$



(f) $\begin{cases} 3 & x \leq 0 \\ 3-x & 0 < x \leq 3 \\ 0 & x > 3 \end{cases}$

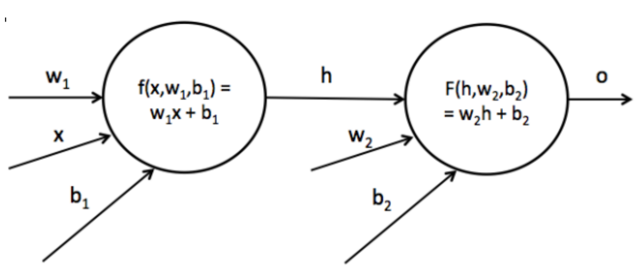


(g) $\log(x)$



(h) $\begin{cases} 0.5x & x \leq 0 \\ 0 & 0 < x \leq 3 \\ 3x-9 & x > 3 \end{cases}$

4. Now we consider neural networks with multiple affine transformations, as indicated below. We now have two sets of weights and biases w_1, b_1 and w_2, b_2. We denote the result of the first transformation h such that $h = w_1 x + b_1$, and $o = w_2 h + b_2$. Which of the functions can be represented by this network? For the options which can, write out appropriate values of w_1, w_2, b_1, b_2.



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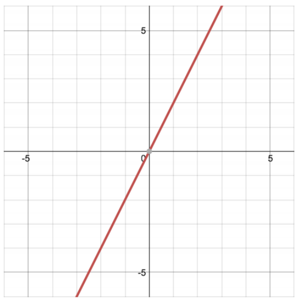
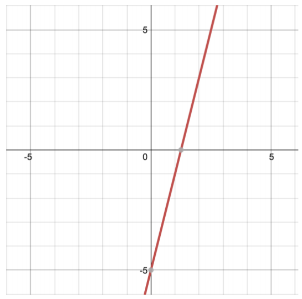
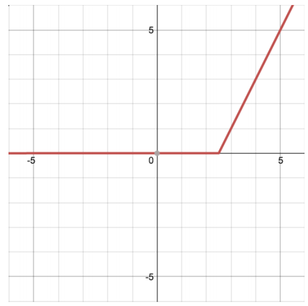


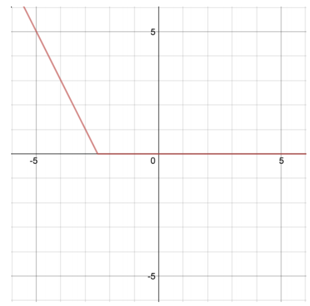
(a) $2x$



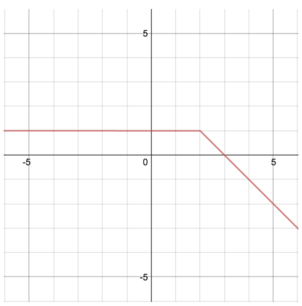
(b) $4x - 5$



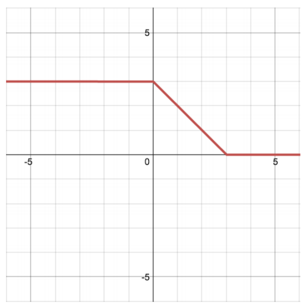
(c) $\begin{cases} 2x - 5 & x \geq 2.5 \\ 0 & x < 2.5 \end{cases}$



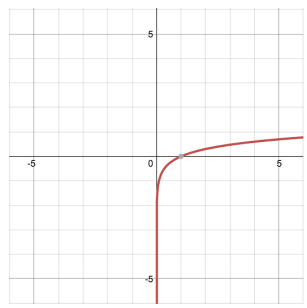
(d) $\begin{cases} -2x - 5 & x \leq -2.5 \\ 0 & x > -2.5 \end{cases}$



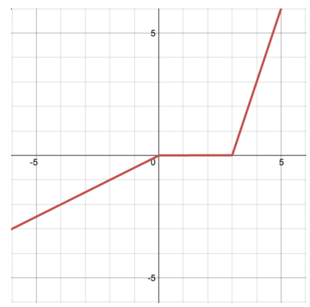
(e) $\begin{cases} -x + 3 & x \geq 2 \\ 1 & x < 2 \end{cases}$



(f) $\begin{cases} 3 & x \leq 0 \\ 3 - x & 0 < x \leq 3 \\ 0 & x > 3 \end{cases}$

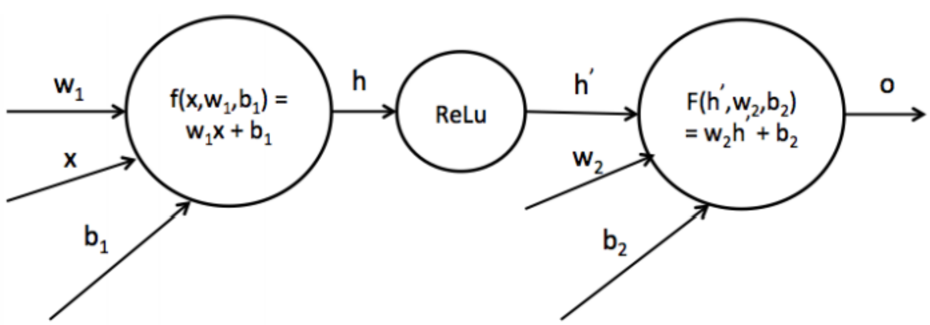


(g) $\log(x)$



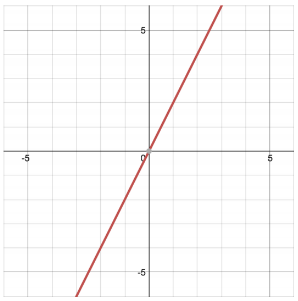
(h) $\begin{cases} 0.5x & x \leq 0 \\ 0 & 0 < x \leq 3 \\ 3x - 9 & x > 3 \end{cases}$

5. Next we add a ReLU non-linearity to the network after the first affine transformation, creating a hidden layer. Which of the functions can be represented by this network? For the options which can, write out appropriate values of w_1, w_2, b_1, b_2.

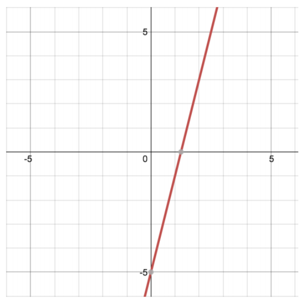


Neural Network Representations

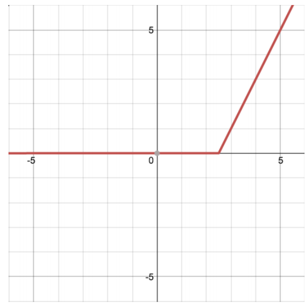
You are given a number of functions (a-h) of a single variable, x, which are graphed below. The computation graphs on the following pages will start off simple and get more complex, building up to neural networks. For each computation graph, indicate which of the functions below they are able to represent.



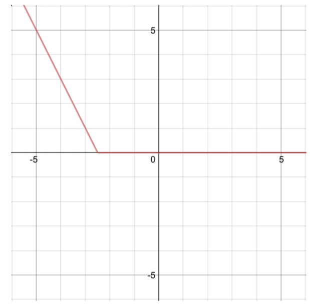
(a) $2x$



(b) $4x - 5$



(c) $\begin{cases} 2x - 5 & x \geq 2.5 \\ 0 & x < 2.5 \end{cases}$



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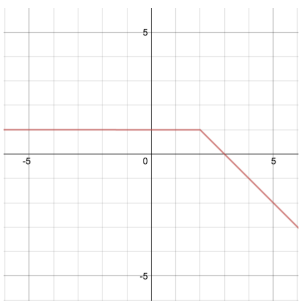
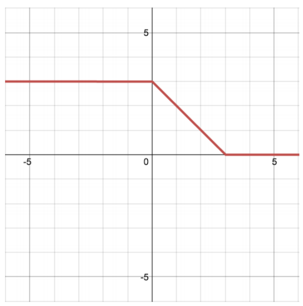
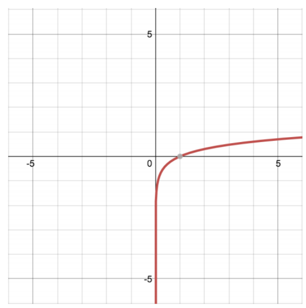


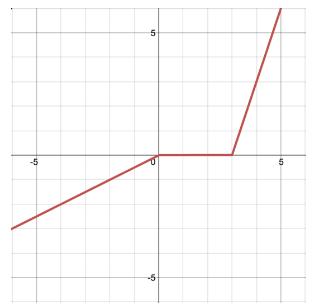
(e) $\begin{cases} -x + 3 & x \geq 2 \\ 1 & x < 2 \end{cases}$



(f) $\begin{cases} 3 & x \leq 0 \\ 3 - x & 0 < x \leq 3 \\ 0 & x > 3 \end{cases}$

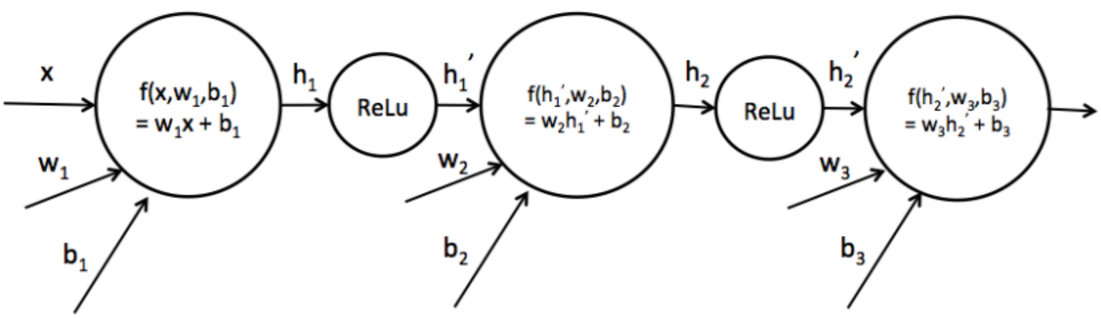


(g) $\log(x)$



(h) $\begin{cases} 0.5x & x \leq 0 \\ 0 & 0 < x \leq 3 \\ 3x - 9 & x > 3 \end{cases}$

6. Now we add another hidden layer to the network, as indicated below. Which of the functions can be represented by this network?



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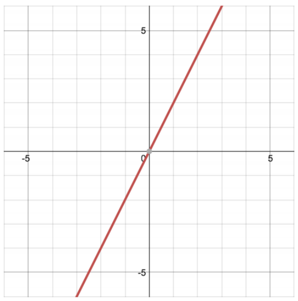
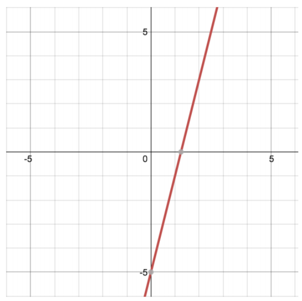
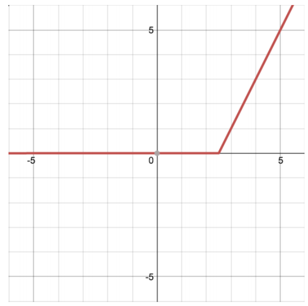


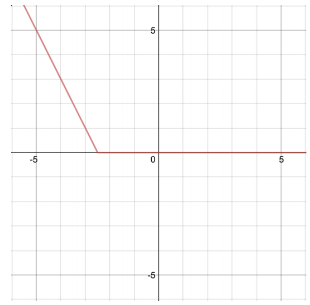
(a) $2x$



(b) $4x - 5$



(c) $\begin{cases} 2x - 5 & x \geq 2.5 \\ 0 & x < 2.5 \end{cases}$



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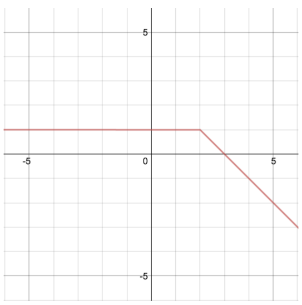
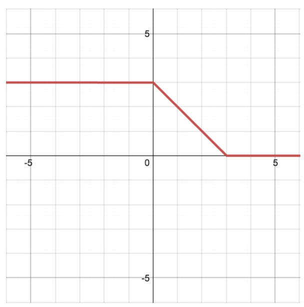
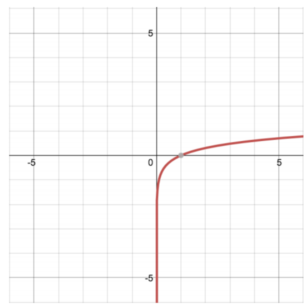


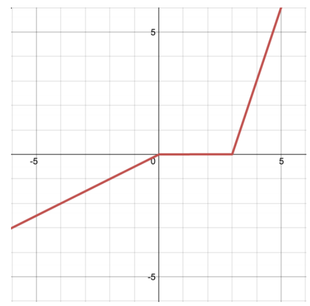
(e) $\begin{cases} -x + 3 & x \geq 2 \\ 1 & x < 2 \end{cases}$



(f) $\begin{cases} 3 & x \leq 0 \\ 3 - x & 0 < x \leq 3 \\ 0 & x > 3 \end{cases}$

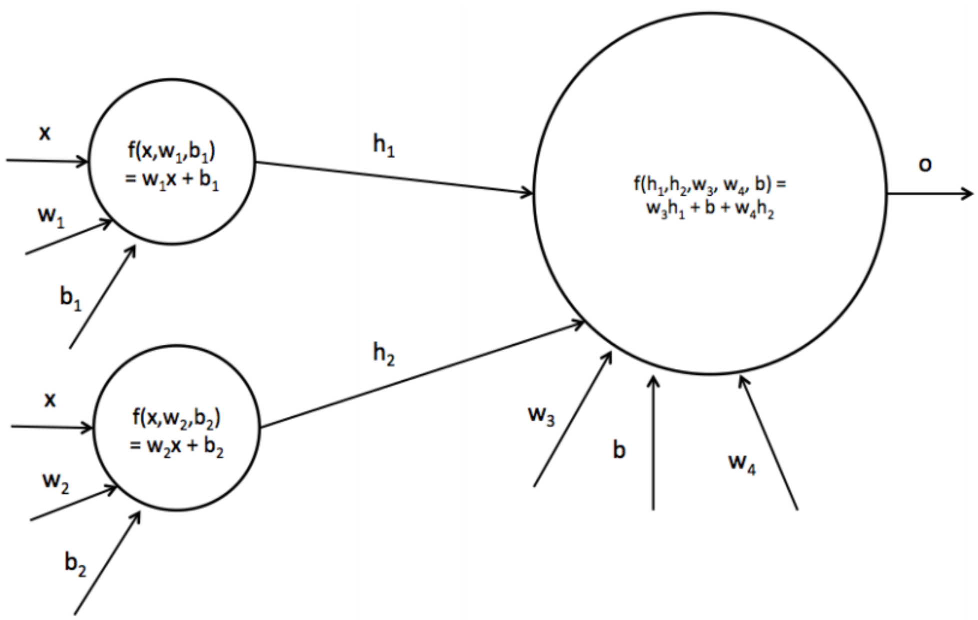


(g) $\log(x)$



(h) $\begin{cases} 0.5x & x \leq 0 \\ 0 & 0 < x \leq 3 \\ 3x - 9 & x > 3 \end{cases}$

7. We'd like to consider using a neural net with just one hidden layer, but have it be larger – a hidden layer of size 2. Let's first consider using just two affine functions, with no nonlinearity in between. Which of the functions can be represented by this network?



Neural Network Representations

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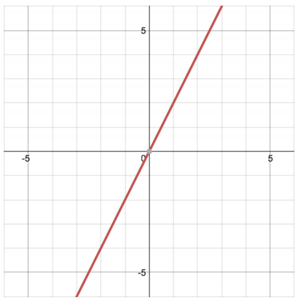
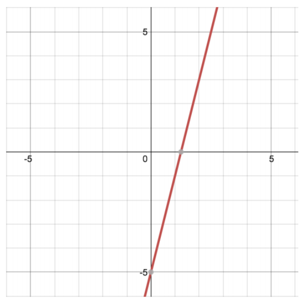
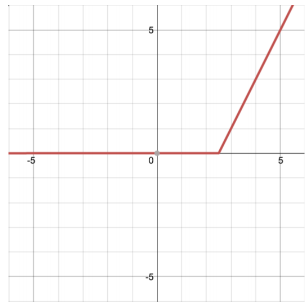


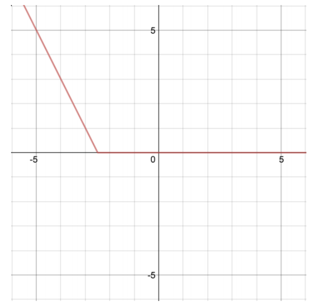
(a) $2x$



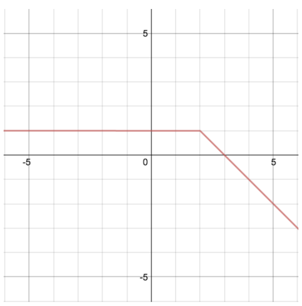
(b) $4x - 5$



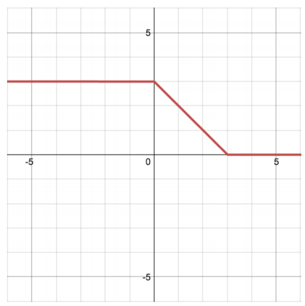
(c) $\begin{cases} 2x - 5 & x \geq 2.5 \\ 0 & x < 2.5 \end{cases}$



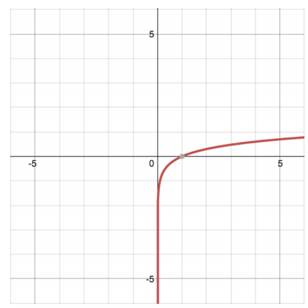
(d) $\begin{cases} -2x - 5 & x \leq -2.5 \\ 0 & x > -2.5 \end{cases}$



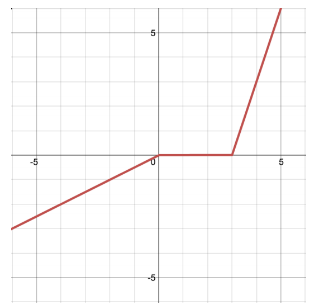
(e) $\begin{cases} -x + 3 & x \geq 2 \\ 1 & x < 2 \end{cases}$



(f) $\begin{cases} 3 & x \leq 0 \\ 3 - x & 0 < x \leq 3 \\ 0 & x > 3 \end{cases}$

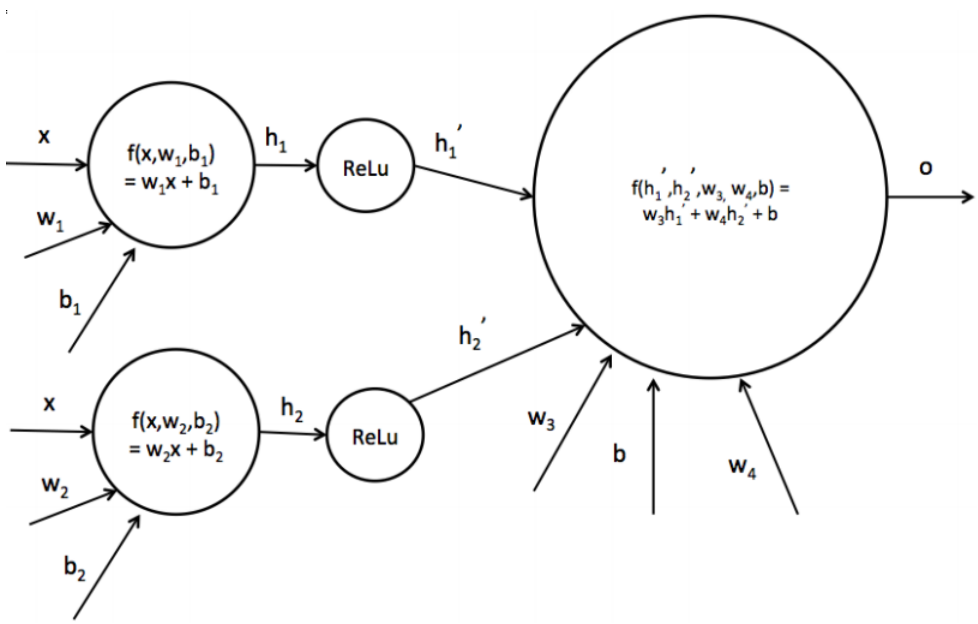


(g) $\log(x)$



(h) $\begin{cases} 0.5x & x \leq 0 \\ 0 & 0 < x \leq 3 \\ 3x - 9 & x > 3 \end{cases}$

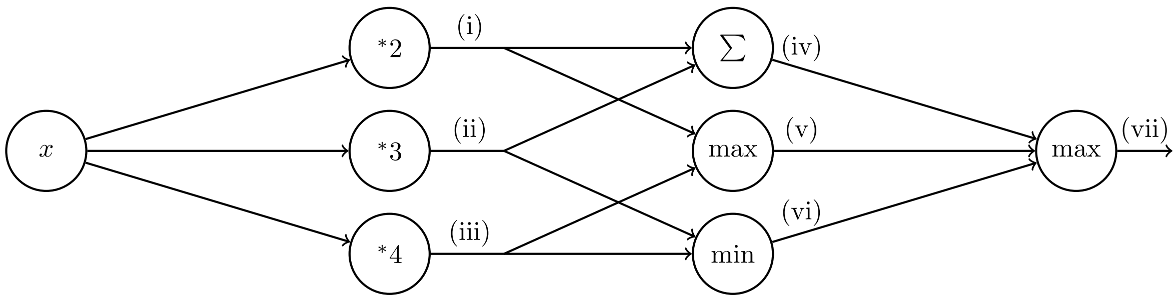
8. Now we'll add a non-linearity between the two affine layers, to produce the neural network below with a hidden layer of size 2. Which of the functions can be represented by this network?



Deep Learning

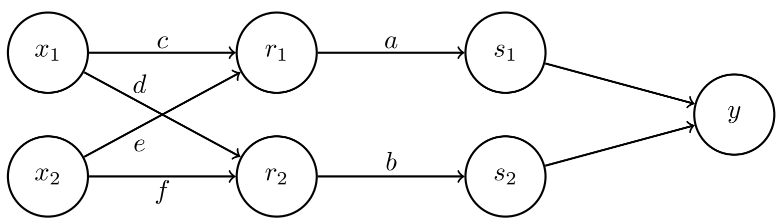
(a) [3 pts] Perform forward propagation on the neural network below for $x = 1$ by filling in the values in the table. Note that (i), ..., (vii) are outputs after performing the appropriate operation as indicated in the node.

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)



(b) [6 pts] Below is a neural network with weights a, b, c, d, e, f. The inputs are x_1 and x_2.
The first hidden layer computes $r_1 = \max(c \cdot x_1 + e \cdot x_2, 0)$ and $r_2 = \max(d \cdot x_1 + f \cdot x_2, 0)$.
The second hidden layer computes $s_1 = \frac{1}{1+\exp(-a \cdot r_1)}$ and $s_2 = \frac{1}{1+\exp(-b \cdot r_2)}$.
The output layer computes $y = s_1 + s_2$. Note that the weights a, b, c, d, e, f are indicated along the edges of the neural network here.

Suppose the network has inputs $x_1 = 1, x_2 = -1$.
The weight values are $a = 1, b = 1, c = 4, d = 1, e = 2, f = 2$.
Forward propagation then computes $r_1 = 2, r_2 = 0, s_1 = 0.9, s_2 = 0.5, y = 1.4$. Note: some values are rounded.

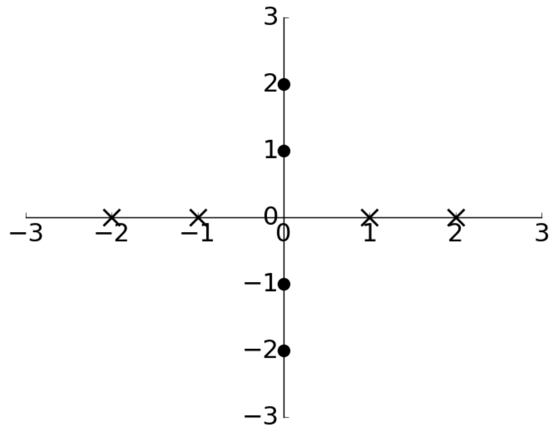


Using the values computed from forward propagation, use backpropagation to numerically calculate the following partial derivatives. Write your answers as a single number (not an expression). You do not need a calculator. Use scratch paper if needed.
Hint: For $g(z) = \frac{1}{1+\exp(-z)}$, the derivative is $\frac{\partial g}{\partial z} = g(z)(1 - g(z))$.

$\frac{\partial y}{\partial a}$	$\frac{\partial y}{\partial b}$	$\frac{\partial y}{\partial c}$	$\frac{\partial y}{\partial d}$	$\frac{\partial y}{\partial e}$	$\frac{\partial y}{\partial f}$

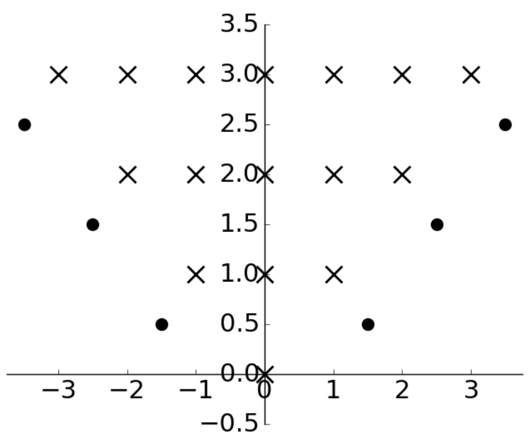
(c) [6 pts] Below are two plots with horizontal axis x_1 and vertical axis x_2 containing data labelled $\times$ and $\bullet$. For each plot, we wish to find a function $f(x_1, x_2)$ such that $f(x_1, x_2) \geq 0$ for all data labelled $\times$ and $f(x_1, x_2) < 0$ for all data labelled $\bullet$.

Below each plot is the function $f(x_1, x_2)$ for that specific plot. Complete the expressions such that all the data is labelled correctly. If not possible, mark "No valid combination".



$f(x_1, x_2) = \max(\underline{\textbf{(i)}} + \underline{\textbf{(ii)}}, \underline{\textbf{(iii)}} + \underline{\textbf{(iv)}}) + \underline{\textbf{(v)}}$

(i) ◯ x_1 ◯ $-x_1$ ◯ 0
(ii) ◯ x_2 ◯ $-x_2$ ◯ 0
(iii) ◯ x_1 ◯ $-x_1$ ◯ 0
(iv) ◯ x_2 ◯ $-x_2$ ◯ 0
(v) ◯ 1 ◯ -1 ◯ 0
◯ No valid combination

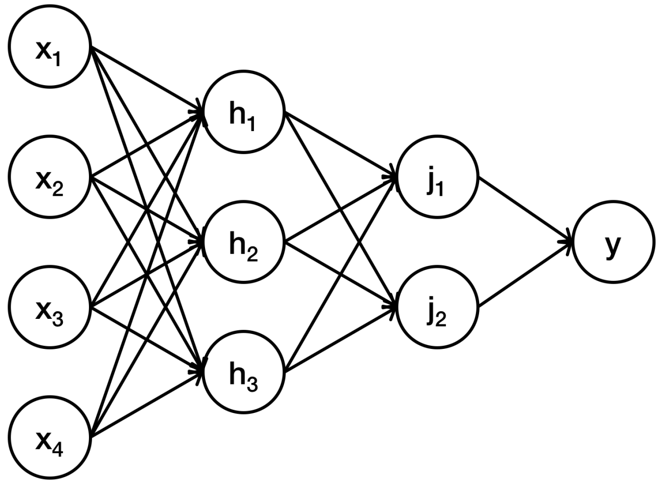


$f(x_1, x_2) = \underline{\textbf{(vi)}} - \max(\underline{\textbf{(vii)}} + \underline{\textbf{(viii)}}, \underline{\textbf{(ix)}} + \underline{\textbf{(x)}})$

(vi) ◯ x_2 ◯ $-x_2$ ◯ 0
(vii) ◯ x_1 ◯ $-x_1$ ◯ 0
(viii) ◯ x_2 ◯ $-x_2$ ◯ 0
(ix) ◯ x_1 ◯ $-x_1$ ◯ 0
(x) ◯ x_2 ◯ $-x_2$ ◯ 0
◯ No valid combination

Q7. [12 pts] Neural Network Data Sufficiency

The next few problems use the below neural network as a reference. Neurons h_{1-3} and j_{1-2} all use ReLU activation functions. Neuron y uses the identity activation function: $f(x) = x$. In the questions below, let $w_{a,b}$ denote the weight that connects neurons a and b. Also, let o_a denote the value that neuron a outputs to its next layer.



Given this network, in the following few problems, you have to decide whether the data given are sufficient for answering the question.

(a) [2 pts] Given the above neural network, what is the value of o_y?

Data item 1: the values of all weights in the network and the values o_{h_1}, o_{h_2}, o_{h_3}
Data item 2: the values of all weights in the network and the values o_{j_1}, o_{j_2}

- ○ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ○ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
- ○ Both statements taken together are sufficient, but neither data item alone is sufficient.
- ● Each data item alone is sufficient to answer the question.
- ○ Statements (1) and (2) together are not sufficient, and additional data is needed to answer the question.

(b) [2 pts] Given the above neural network, what is the value of o_{h_1}?

Data item 1: the neuron input values, i.e., o_{x_1} through o_{x_4}
Data item 2: the values o_{j_1}, o_{j_2}

- ○ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ○ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
- ○ Both statements taken together are sufficient, but neither data item alone is sufficient.
- ○ Each data item alone is sufficient to answer the question.
- ● Statements (1) and (2) together are not sufficient, and additional data is needed to answer the question.

(c) [2 pts] Given the above neural network, what is the value of o_{j_1}?

Data item 1: the values of all weights connecting neurons h_1, h_2, h_3 to j_1, j_2
Data item 2: the values o_{h_1}, o_{h_2}, o_{h_3}

- ○ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ○ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
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(d) [2 pts] Given the above neural network, what is the value of $\partial o_y / \partial w_{j_2,y}$?

Data item 1: the value of o_{j_2}
Data item 2: all weights in the network and the neuron input values, i.e., o_{x_1} through o_{x_4}

- ○ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ○ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
- ○ Both statements taken together are sufficient, but neither data item alone is sufficient.
- ● Each data item alone is sufficient to answer the question.
- ○ Statements (1) and (2) together are not sufficient, and additional data is needed to answer the question.

(e) [2 pts] Given the above neural network, what is the value of $\partial o_y / \partial w_{h_2,j_2}$?

Data item 1: the value of $w_{j_2,y}$
Data item 2: the value of $\partial o_{j_2} / \partial w_{h_2,j_2}$

- ○ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ○ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
- ● Both statements taken together are sufficient, but neither data item alone is sufficient.
- ○ Each data item alone is sufficient to answer the question.
- ○ Statements (1) and (2) together are not sufficient, and additional data is needed to answer the question.

(f) [2 pts] Given the above neural network, what is the value of $\partial o_y / \partial w_{x_1,h_3}$?

Data item 1: the value of all weights in the network and the neuron input values, i.e., o_{x_1} through o_{x_4}
Data item 2: the value of w_{x_1,h_3}

- ● Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ○ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
- ○ Both statements taken together are sufficient, but neither data item alone is sufficient.
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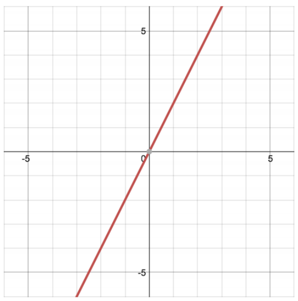
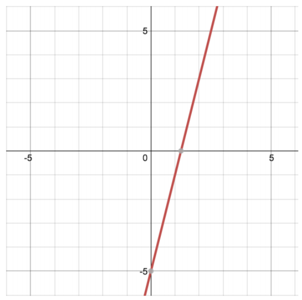
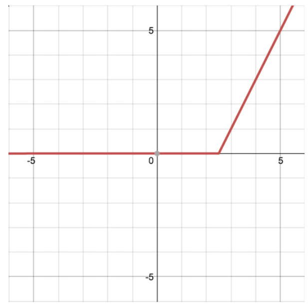


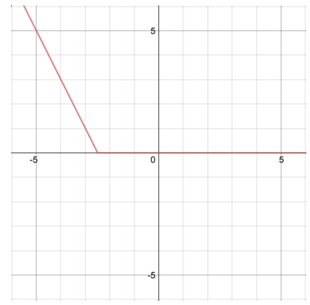
(a) $2x$



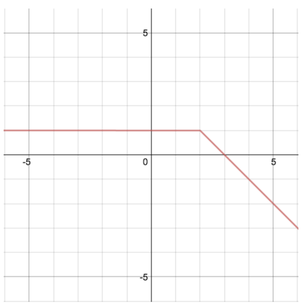
(b) $4x - 5$



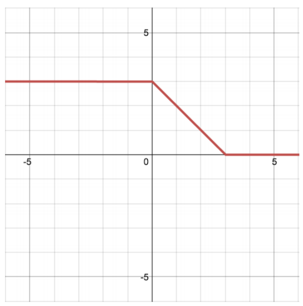
(c) $\begin{cases} 2x - 5 & x \geq 2.5 \\ 0 & x < 2.5 \end{cases}$



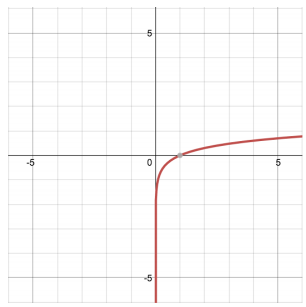
(d) $\begin{cases} -2x - 5 & x \leq -2.5 \\ 0 & x > -2.5 \end{cases}$



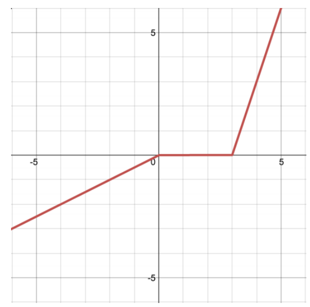
(e) $\begin{cases} -x + 3 & x \geq 2 \\ 1 & x < 2 \end{cases}$



(f) $\begin{cases} 3 & x \leq 0 \\ 3 - x & 0 < x \leq 3 \\ 0 & x > 3 \end{cases}$

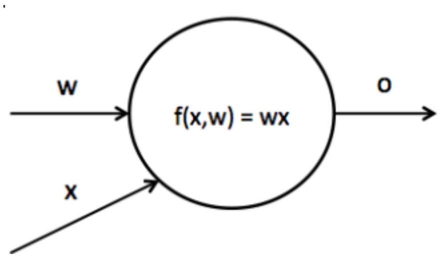


(g) $\log(x)$



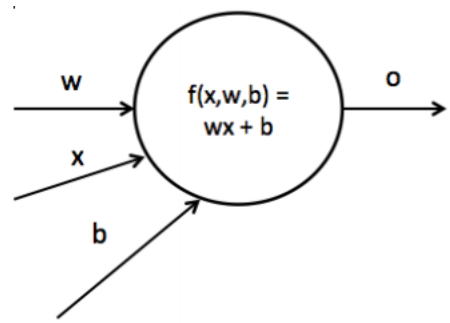
(h) $\begin{cases} 0.5x & x \leq 0 \\ 0 & 0 < x \leq 3 \\ 3x - 9 & x > 3 \end{cases}$

1. Consider the following computation graph, computing a linear transformation with scalar input x, weight w, and output o, such that $o = wx$. Which of the funcions can be represented by this graph? For the options which can, write out the appropriate value of w.



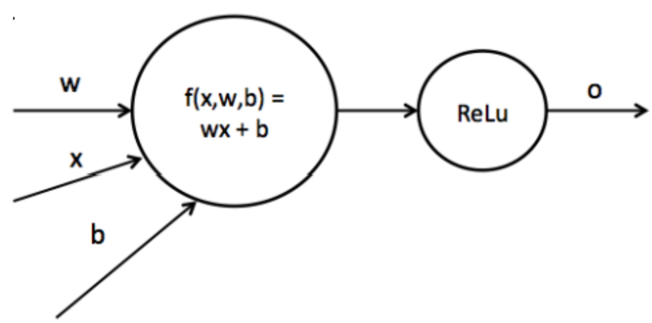
This graph can only represent (a), with $w = 2$. Since there is no bias term, the line must pass through the origin.

2. Now we introduce a bias term b into the graph, such that $o = wx + b$ (this is known as an *affine* function). Which of the functions can be represented by this network? For the options which can, write out an appropriate value of w, b.



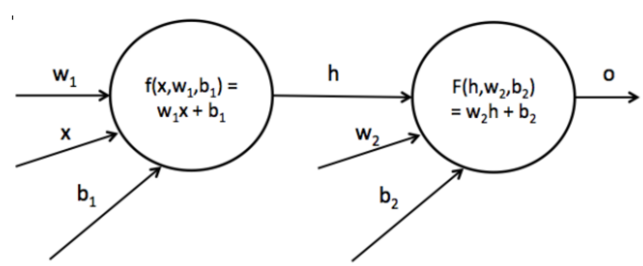
(a) with $w = 2$ and $b = 0$, and (b) with $w = 4$ and $b = -5$

3. We can introduce a non-linearity into the network as indicated below. We use the ReLU non-linearity, which has the form $ReLU(x) = \max(0, x)$. Now which of the functions can be represented by this neural network with weight w and bias b? For the options which can, write out an appropriate value of w, b.



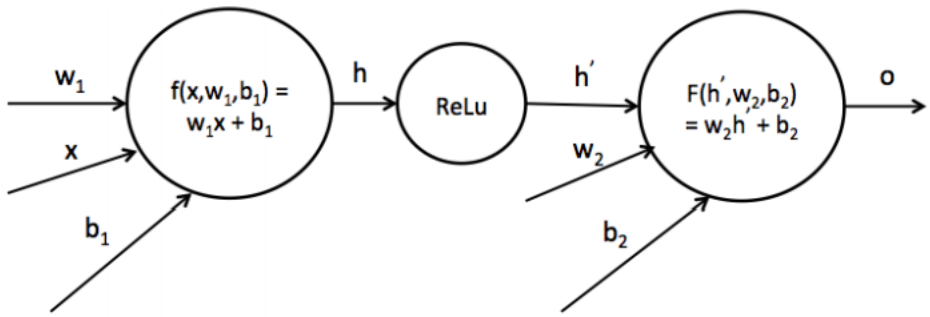
With the output coming directly from the ReLU, this cannot produce any values less than zero. It can produce (c) with $w = 2$ and $b = -5$, and (d) with $w = -2$ and $b = -5$

4. Now we consider neural networks with multiple affine transformations, as indicated below. We now have two sets of weights and biases w_1, b_1 and w_2, b_2. We denote the result of the first transformation h such that $h = w_1 x + b_1$, and $o = w_2 h + b_2$. Which of the functions can be represented by this network? For the options which can, write out appropriate values of w_1, w_2, b_1, b_2.



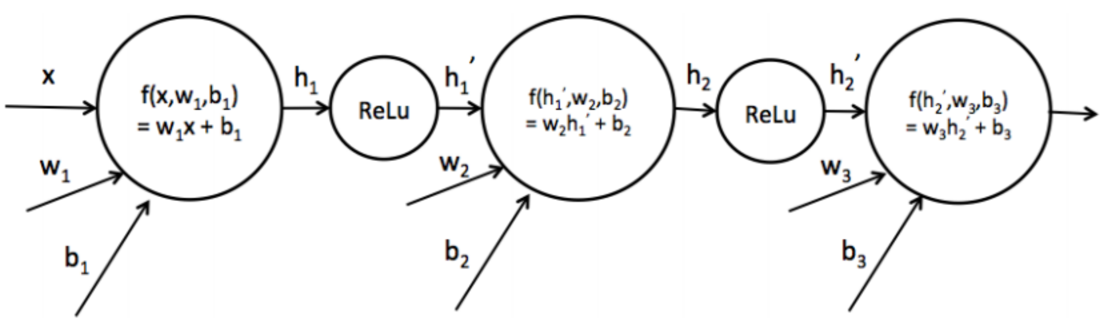
Applying multiple affine transformations (with no non-linearity in between) is not any more powerful than a single affine function: $w_2(w_1 x + b_1) + b_2 = w_2 w_1 x + w_2 b_1 + b_2$, so this is just a affine function with different coefficients. The functions we can represent are the same as in 1, if we choose $w_1 = w, w_2 = 0, b_1 = 0, b_2 = b$: (a) with $w_1 = 2, w_2 = 1, b_1 = 0, b_2 = 0$, and (b) with $w_1 = 4, w_2 = 1, b_1 = 0, b_2 = -5$.

5. Next we add a ReLU non-linearity to the network after the first affine transformation, creating a hidden layer. Which of the functions can be represented by this network? For the options which can, write out appropriate values of w_1, w_2, b_1, b_2.



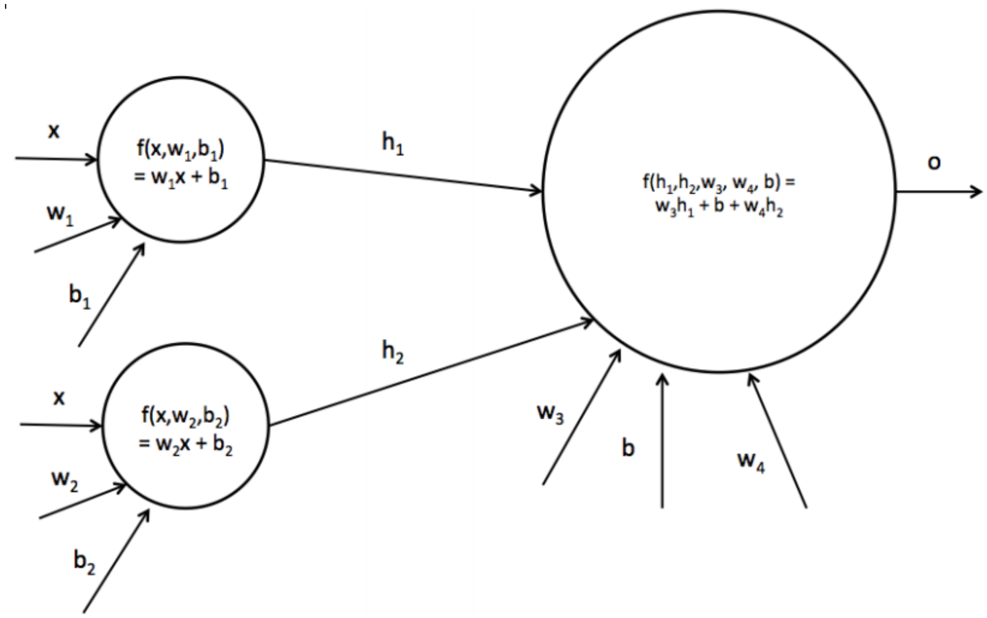
(c), (d), and (e). The affine transformation after the ReLU is capable of stretching (or flipping) and shifting the ReLU output in the vertical dimension. The parameters to produce these are: (c) with $w_1 = 2, b_1 = -5, w_2 = 1, b_2 = 0$, (d) with $w_1 = -2, b_1 = -5, w_2 = 1, b_2 = 0$, and (e) with $w_1 = 1, b_1 = -2, w_2 = -1, b_2 = 1$

6. Now we add another hidden layer to the network, as indicated below. Which of the functions can be represented by this network?



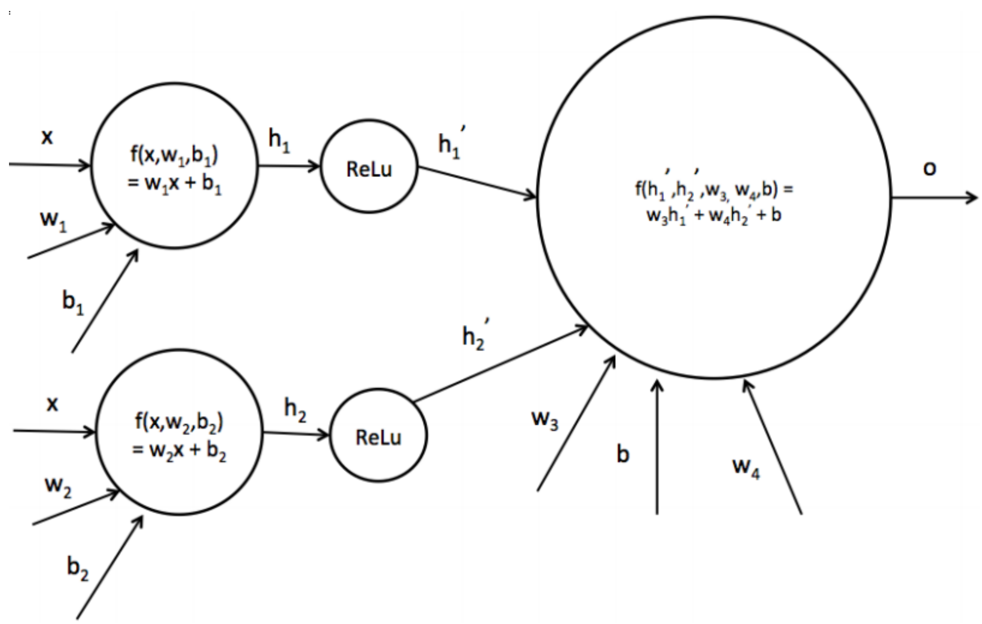
(c), (d), (e), and (f). The network can represent all the same functions as Q5 (because note that we could have $w_2 = 1$ and $b_2 = 0$). In addition it can represent (f): the first ReLU can produce the first flat segment, the affine transformation can flip and shift the resulting curve, and then the second ReLU can produce the second flat segment (with the final affine layer not doing anything). Note that (h) cannot be produced since its line has only one flat segment (and the affine layers can only scale, shift, and flip the graph in the vertical dimension; they can't rotate the graph).

7. We'd like to consider using a neural net with just one hidden layer, but have it be larger – a hidden layer of size 2. Let's first consider using just two affine functions, with no nonlinearity in between. Which of the functions can be represented by this network?



(a) and (b). With no non-linearity, this reduces to a single affine function (in the same way as Q4)

8. Now we'll add a non-linearity between the two affine layers, to produce the neural network below with a hidden layer of size 2. Which of the functions can be represented by this network?

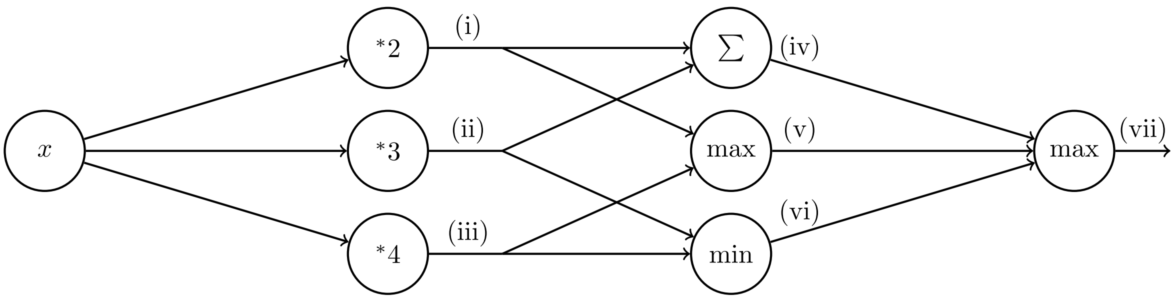


All functions except for (g). Note that we can recreate any network from (5) by setting w_4 to 0, so this allows us to produce (c), (d) and (e). To produce the rest of the functions, note that h_1' and h_2' will be two independent functions with a flat part lying on the x-axis, and a portion with positive slope. The final layer takes a weighted sum of these two functions. To produce (a) and (b), the flat portion of one ReLU should start at the point where the other ends ($x = 0$ for (a), or $x = 1$ for (b). The final layer then vertically flips the ReLU sloping down and adds it to the one sloping up, producing a single sloped line. To produce (h), the ReLU sloping down should have its flat portion end (at $x = 0$ before the other's flat portion begins (at $x = 3$). The down-sloping one is again flipped and added to the up-sloping. To produce (f), both ReLUs should have equal slope, which will cancel to produce the first flat portion above the x-axis.

Q4. [15 pts] Deep Learning

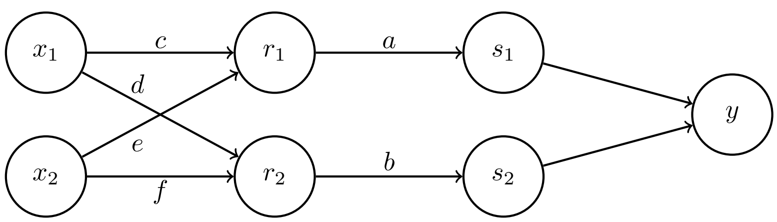
(a) [3 pts] Perform forward propagation on the neural network below for $x = 1$ by filling in the values in the table. Note that (i), …, (vii) are outputs after performing the appropriate operation as indicated in the node.

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
2	3	4	5	4	3	5



(b) [6 pts] Below is a neural network with weights a, b, c, d, e, f. The inputs are x_1 and x_2.
The first hidden layer computes $r_1 = \max(c \cdot x_1 + e \cdot x_2, 0)$ and $r_2 = \max(d \cdot x_1 + f \cdot x_2, 0)$.
The second hidden layer computes $s_1 = \frac{1}{1+\exp(-a \cdot r_1)}$ and $s_2 = \frac{1}{1+\exp(-b \cdot r_2)}$.
The output layer computes $y = s_1 + s_2$. Note that the weights a, b, c, d, e, f are indicated along the edges of the neural network here.

Suppose the network has inputs $x_1 = 1, x_2 = -1$.
The weight values are $a = 1, b = 1, c = 4, d = 1, e = 2, f = 2$.
Forward propagation then computes $r_1 = 2, r_2 = 0, s_1 = 0.9, s_2 = 0.5, y = 1.4$. Note: some values are rounded.



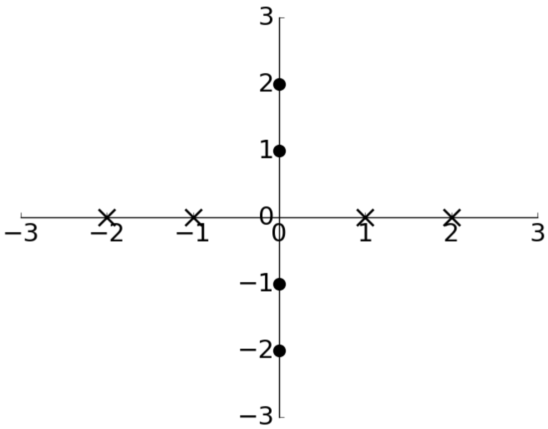
Using the values computed from forward propagation, use backpropagation to numerically calculate the following partial derivatives. Write your answers as a single number (not an expression). You do not need a calculator. Use scratch paper if needed.
Hint: For $g(z) = \frac{1}{1+\exp(-z)}$, the derivative is $\frac{\partial g}{\partial z} = g(z)(1 - g(z))$.

$\frac{\partial y}{\partial a}$	$\frac{\partial y}{\partial b}$	$\frac{\partial y}{\partial c}$	$\frac{\partial y}{\partial d}$	$\frac{\partial y}{\partial e}$	$\frac{\partial y}{\partial f}$
0.18	0	0.09	0	-0.09	0

$$
\begin{aligned}
\frac{\partial y}{\partial a} &= \frac{\partial y}{\partial s_1}\frac{\partial s_1}{\partial a} \\
&= 1 \cdot \frac{\partial g(a \cdot r_1)}{\partial a} \\
&= r_1 \cdot g(a \cdot r_1)(1 - g(a \cdot r_1)) \\
&= r_1 \cdot s_1(1 - s_1) \\
&= 2 \cdot 0.9 \cdot (1 - 0.9) \\
&= 0.18 \\
\frac{\partial y}{\partial b} &= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial b} \\
&= 1 \cdot \frac{\partial g(b \cdot r_2)}{\partial b} \\
&= r_2 \cdot g(b \cdot r_2)(1 - g(b \cdot r_2)) \\
&= r_2 \cdot s_2(1 - s_2) \\
&= 0 \cdot 0.5(1 - 0.5) \\
&= 0 \\
\frac{\partial y}{\partial c} &= \frac{\partial y}{\partial s_1}\frac{\partial s_1}{\partial r_1}\frac{\partial r_1}{\partial c} \\
&= 1 \cdot [a \cdot g(a \cdot r_1)(1 - g(a \cdot r_1))] \cdot x_1 \\
&= [a \cdot s_1(1 - s_1)] \cdot x_1 \\
&= [1 \cdot 0.9(1 - 0.9)] \cdot 1 \\
&= 0.09 \\
\frac{\partial y}{\partial d} &= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial r_2}\frac{\partial r_2}{\partial d} \\
&= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial r_2} \cdot 0 \\
&= 0 \\
\frac{\partial y}{\partial e} &= \frac{\partial y}{\partial s_1}\frac{\partial s_1}{\partial r_1}\frac{\partial r_1}{\partial e} \\
&= 1 \cdot [a \cdot g(a \cdot r_1)(1 - g(a \cdot r_1))] \cdot x_2 \\
&= [a \cdot s_1(1 - s_1)] \cdot x_2 \\
&= [1 \cdot 0.9(1 - 0.9)] \cdot -1 \\
&= -0.09 \\
\frac{\partial y}{\partial f} &= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial r_2}\frac{\partial r_2}{\partial f} \\
&= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial r_2} \cdot 0 \\
&= 0
\end{aligned}
$$

(c) [6 pts] Below are two plots with horizontal axis x_1 and vertical axis x_2 containing data labelled $\times$ and $\bullet$. For each plot, we wish to find a function $f(x_1, x_2)$ such that $f(x_1, x_2) \geq 0$ for all data labelled $\times$ and $f(x_1, x_2) < 0$ for all data labelled $\bullet$.

Below each plot is the function $f(x_1, x_2)$ for that specific plot. Complete the expressions such that all the data is labelled correctly. If not possible, mark "No valid combination".



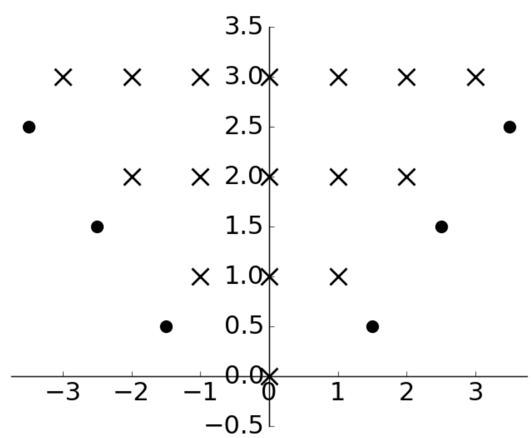
$f(x_1, x_2) = \max(\underline{\textbf{(i)}} + \underline{\textbf{(ii)}}, \underline{\textbf{(iii)}} + \underline{\textbf{(iv)}}) + \underline{\textbf{(v)}}$

(i)	● x_1	○ $-x_1$	○ 0
(ii)	○ x_2	○ $-x_2$	● 0
(iii)	○ x_1	● $-x_1$	○ 0
(iv)	○ x_2	○ $-x_2$	● 0
(v)	○ 1	● -1	○ 0

○ No valid combination

There are two possible solutions:

$$f(x_1, x_2) = \max(x_1, -x_1) - 1$$
$$f(x_1, x_2) = \max(-x_1, x_1) - 1$$



$f(x_1, x_2) = \underline{\textbf{(vi)}} - \max(\underline{\textbf{(vii)}} + \underline{\textbf{(viii)}}, \underline{\textbf{(ix)}} + \underline{\textbf{(x)}})$

(vi)	● x_2	○ $-x_2$	○ 0
(vii)	● x_1	○ $-x_1$	○ 0
(viii)	○ x_2	○ $-x_2$	● 0
(ix)	○ x_1	● $-x_1$	○ 0
(x)	○ x_2	○ $-x_2$	● 0

○ No valid combination

There are four possible solutions:

$$f(x_1, x_2) = x_2 - \max(x_1, -x_1)$$
$$f(x_1, x_2) = x_2 - \max(-x_1, x_1)$$
$$f(x_1, x_2) = -\max(x_1 - x_2, -x_1 - x_2)$$
$$f(x_2, x_2) = -\max(-x_1 - x_2, x_1 - x_2)$$