

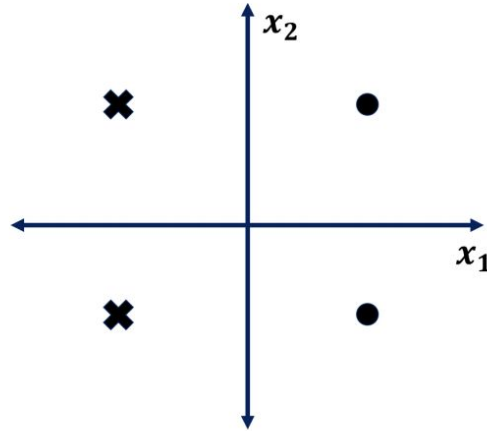
# Perceptrons Exercise\*

\*adapted from UCB Su19 final

For each of the datasets represented by the graphs below, please select the feature maps for which the perceptron algorithm can perfectly classify the data.

Each data point is in the form  $(x_1, x_2)$ , and has some label  $Y$ , which is either a 1 (dot) or  $-1$  (cross).

(i) [5 pts]



$[x_1 \ x_2 \ 1]$

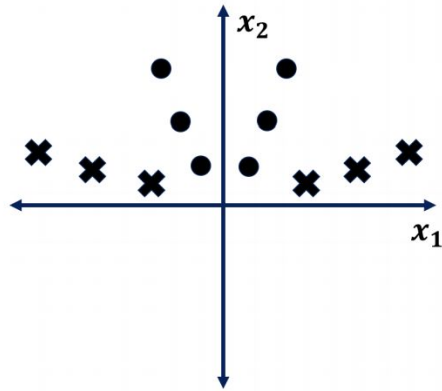
$[x_1 \ x_2 \ x_1^2]$

$[x_1 \ x_2 \ |x_1|]$

$[x_1 \ x_2 \ Y]$

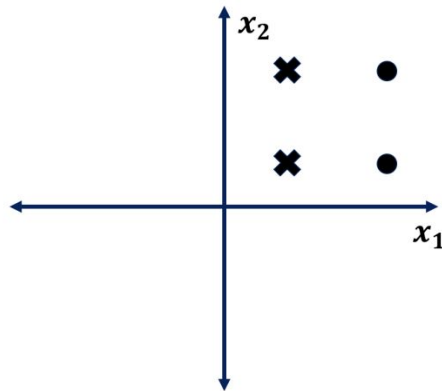
$[x_1 \ x_2]$

(ii) [5 pts]



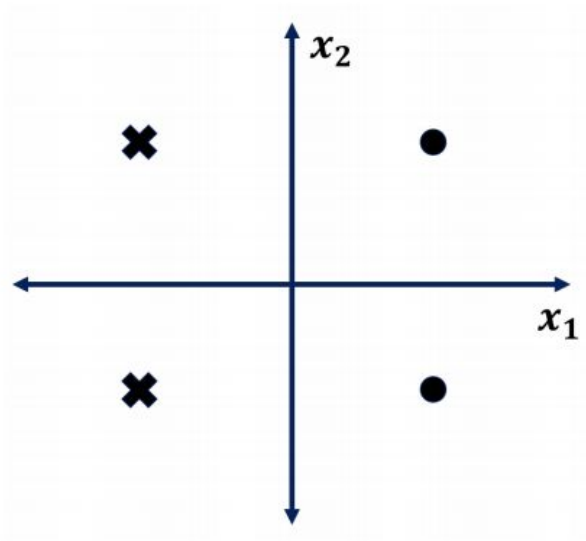
- $[x_1 \ x_2 \ 1]$
- $[x_1 \ x_2 \ x_1^2]$
- $[x_1 \ x_2 \ |x_1|]$
- $[x_1 \ x_2 \ Y]$
- $[x_1 \ x_2]$

(iii) [5 pts]



- $[x_1 \ x_2 \ 1]$
- $[x_1 \ x_2 \ x_1^2]$
- $[x_1 \ x_2 \ |x_1|]$
- $[x_1 \ x_2 \ Y]$
- $[x_1 \ x_2]$

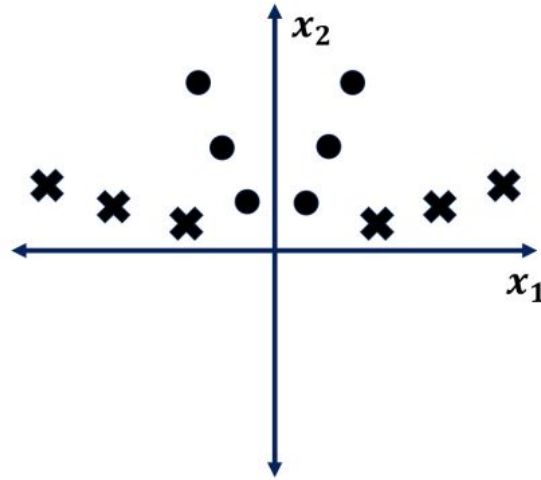
(i) [5 pts]



- $[x_1 \ x_2 \ 1]$
- $[x_1 \ x_2 \ x_1^2]$
- $[x_1 \ x_2 \ |x_1|]$
- $[x_1 \ x_2 \ Y]$
- $[x_1 \ x_2]$

The line  $x_1 = 0$  allows us to linearly separate the data. Since  $x_1$  is a feature in all of the choices, all choices are valid. We simply ignore all of the other features by setting their weight to 0.

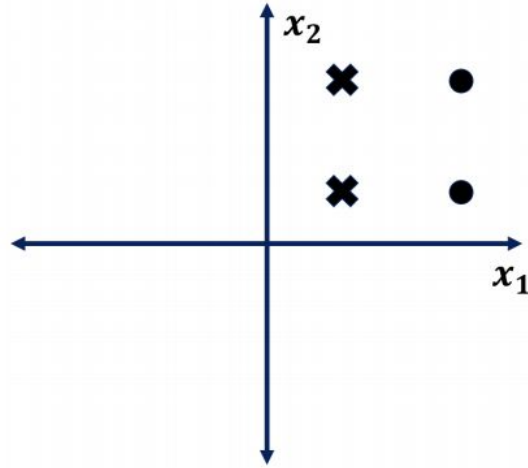
(ii) [5 pts]



- $[x_1 \ x_2 \ 1]$
- $[x_1 \ x_2 \ x_1^2]$
- $[x_1 \ x_2 \ |x_1|]$
- $[x_1 \ x_2 \ Y]$
- $[x_1 \ x_2]$

- Choice A: These features only allow us to draw shifted lines in the  $x_1$ - $x_2$  plane, and no such line can separate the data.
- Choice B: The line  $x_2 = c * x_1^2$  can linearly separate the data, for some constant  $c$ .
- Choice C: The line  $x_2 = c * |x_1|$  can linearly separate the data, for some constant  $c$ .
- Choice D: The line  $Y = 0$  can linearly separate the data.
- Choice E: These features only allow us to draw lines centered at the origin in the  $x_1$ - $x_2$  plane, and no such line can separate the data.

(iii) [5 pts]



- $[x_1 \ x_2 \ 1]$
- $[x_1 \ x_2 \ x_1^2]$
- $[x_1 \ x_2 \ |x_1|]$
- $[x_1 \ x_2 \ Y]$
- $[x_1 \ x_2]$

- Choice A: The line  $x_1 = c * (1)$  can linearly separate the data, for some constant  $c$ .
- Choice B: The line  $x_1^2 = c * x_1$  can linearly separate the data, for some constant  $c$ .
- Choice C: No linear combination of these features allows us to separate the data.
- Choice D: The line  $Y = 0$  can linearly separate the data.
- Choice E: These features only allow us to draw lines centered at the origin in the  $x_1$ - $x_2$  plane, and no such line can separate the data.