CS 188: Artificial Intelligence

CSPs II + Local Search

Profs. Peter Stone and Yuke Zhu

The University of Texas at Austin

[These slides based on those of Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]
Announcements

- **Instructor AMA Office Hour**
  - Come to have your non class-related questions answered
    - Career paths, AI research, AI industry, ...
  - First one: Wednesday 2/10, 3-4 pm.
  - Second one: after the Spring Break
Last time: CSPs

- CSPs:
  - Variables
  - Domains
  - Constraints
    - Implicit (provide code to compute)
    - Explicit (provide a list of the legal tuples)
    - Unary / Binary / N-ary

- Goals:
  - Here: find any solution
  - Also: find all, find best, etc.
A simple form of propagation makes sure all arcs are consistent:

Important: If X loses a value, neighbors of X need to be rechecked!
Arc consistency detects failure earlier than forward checking
Can be run as a preprocessor or after each assignment
What’s the downside of enforcing arc consistency?

Remember: Delete from the tail!
Limitations of Arc Consistency

- After enforcing arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)

- Arc consistency still runs inside a backtracking search!

What went wrong here?
Improving Backtracking

- General-purpose ideas give huge gains in speed
  - ... but it’s all still NP-hard

- Filtering: Can we detect inevitable failure early?

- Ordering:
  - Which variable should be assigned next? (MRV)
  - In what order should its values be tried? (LCV)

- Structure: Can we exploit the problem structure?
Exercise: Arc Consistency + Ordering
Extreme case: independent subproblems
- Example: Tasmania and mainland do not interact

Independent subproblems are identifiable as connected components of constraint graph

Suppose a graph of $n$ variables can be broken into subproblems of only $c$ variables:
- Worst-case solution cost is $O((n/c)(d^c))$, linear in $n$
- E.g., $n = 80$, $d = 2$, $c = 20$
- $2^{80} = 4$ billion years at 10 million nodes/sec
- $(4)(2^{20}) = 0.4$ seconds at 10 million nodes/sec
Tree-Structured CSPs

- Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n d^2)$ time
  - Compare to general CSPs, where worst-case time is $O(d^n)$
- This property also applies to probabilistic reasoning (later): an example of the relation between syntactic restrictions and the complexity of reasoning
Tree-Structured CSPs

- Algorithm for tree-structured CSPs:
  - Order: Choose a root variable, order variables so that parents precede children
  - Remove backward: For $i = n : 2$, apply RemoveInconsistent(\(\text{Parent}(X_i), X_i\))
  - Assign forward: For $i = 1 : n$, assign $X_i$ consistently with \(\text{Parent}(X_i)\)

- Runtime: $O(n d^2)$ (why?)
Claim 1: After backward pass, all root-to-leaf arcs are consistent
Proof: Each $X \rightarrow Y$ was made consistent at one point and $Y$’s domain could not have been reduced thereafter (because $Y$’s children were processed before $Y$)

Claim 2: If root-to-leaf arcs are consistent, forward assignment will not backtrack
Proof: Induction on position

Why doesn’t this algorithm work with cycles in the constraint graph?

Note: we’ll see this basic idea again with Bayes’ nets
Nearly Tree-Structured CSPs

- **Conditioning**: instantiate a variable, prune its neighbors' domains
- **Cutset conditioning**: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree
- **Cutset size c** gives runtime $O\left(d^c(n-c)d^2\right)$, very fast for small $c$
Cutset Conditioning

Choose a cutset

Instantiate the cutset (all possible ways)

Compute residual CSP for each assignment

Solve the residual CSPs (tree structured), removing any inconsistent domain values w.r.t. cutset assignment

\[ (n-c)d^2 \]
Exercise: Cutset Exercise
Iterative Algorithms for CSPs

- Local search methods typically work with “complete” states, i.e., all variables assigned

- To apply to CSPs:
  - Take an assignment with unsatisfied constraints
  - Operators *reassign* variable values
  - No fringe! Live on the edge.

- Algorithm: While not solved,
  - Variable selection: randomly select any conflicted variable
  - Value selection: min-conflicts heuristic:
    - Choose a value that violates the fewest constraints
    - I.e., hill climb with \( h(n) = \text{total number of violated constraints} \)

- Can get stuck in local minima (we’ll come back to this idea in a few slides)
Example: 4-Queens

- States: 4 queens in 4 columns ($4^4 = 256$ states)
- Operators: move queen in column
- Goal test: no attacks
- Evaluation: $c(n) =$ number of attacks
Performance of Min-Conflicts

- Runtime of min-conflicts is on n-queens is **roughly independent of problem size**!
  - Why?? Solutions are densely distributed in state space

- Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000) in ~50 steps!

- The same appears to be true for any randomly-generated CSP except in a narrow range of the ratio

\[
R = \frac{\text{number of constraints}}{\text{number of variables}}
\]
Summary: CSPs

- CSPs are a special kind of search problem:
  - States are partial assignments
  - Goal test defined by constraints

- Basic solution: backtracking search

- Speed-ups:
  - Ordering
  - Filtering
  - Structure

- Iterative min-conflicts is often effective in practice
Local Search

- Tree search keeps unexplored alternatives on the fringe (ensures completeness)

- Local search: improve a single option until you can’t make it better (no fringe!)

- New successor function: local changes

- Generally much faster and more memory efficient (but incomplete and suboptimal)
Hill Climbing

- Simple, general idea:
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit

- What’s bad about this approach?
  - Complete?
  - Optimal?

- What’s good about it?
Hill Climbing Quiz
Simulated Annealing

- **Idea:** Escape local maxima by allowing downhill moves
  - But make them rarer as time goes on

```plaintext
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
    inputs: problem, a problem
             schedule, a mapping from time to "temperature"
    local variables: current, a node
                     next, a node
                     T, a "temperature" controlling prob. of downward steps

    current ← MAKE-NODE(INITIAL-STATE[problem])
    for t ← 1 to ∞ do
        T ← schedule[t]
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← VALUE[next] - VALUE[current]
        if ΔE > 0 then current ← next
        else current ← next only with probability \( e^{ΔE/T} \)
```

Shake! Shake!
Genetic algorithms use a natural selection metaphor
- Keep best N hypotheses at each step (selection) based on a fitness function
- Also have pairwise crossover operators, with optional mutation to give variety

Possibly the most misunderstood, misapplied (and even maligned) technique around
Example: N-Queens

- Why does crossover make sense here?
- When wouldn’t it make sense?
- What would mutation be?
- What would a good fitness function be?
Next Time: Adversarial Search!